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Rolling regular-faced polyhedron on n-uniform tilings

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Master thesis presentation 2020-2021

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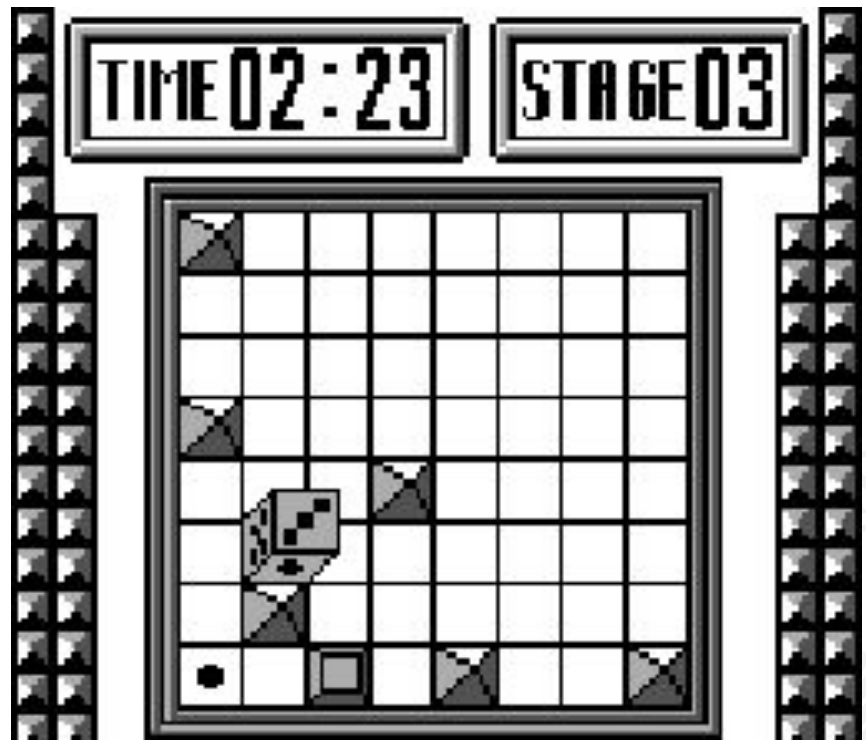
Presentation Structure

1. What is a roller?
   a. Rolling cube
   b. Polyhedron
   c. Tessellation
2. Rolling on a tessellation
   a. Notable results (video)
   b. Rolling patterns results
   c. Desirable traits
3. Future research and Conclusion

What’s rolling a shape?

Cubes on square grid have the monopoly on rolling puzzles



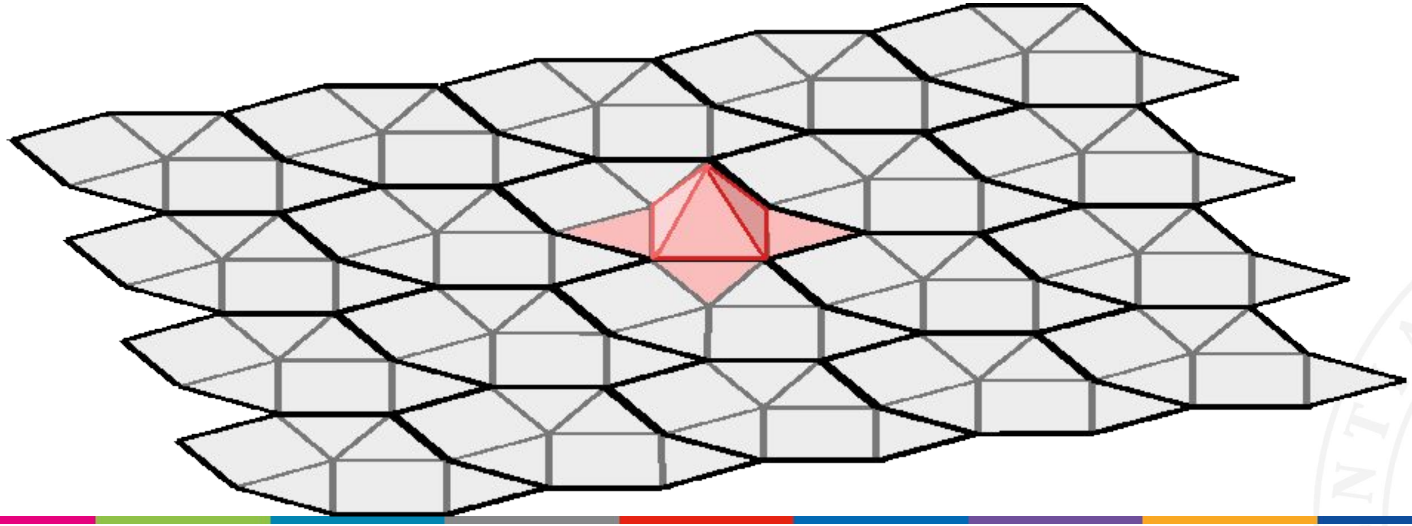
Cube rolling: well known

- Can start from anywhere on the grid
- Can reach everywhere on the grid
  - with any face (face-complete)
  - with two orientations (180°)

Other rolling: unknown

- Some research on tetrahedron
- Some known rolling pairs (dices)
- Tessellation polyhedron (motivation)

What if non-cube... → New research!



Other rolling: research

What would it take?

Shape: Polyhedron with polygon faces

Grid: Cover the plane using polygons

(=tessellations / tiling)

Roll on the plane, look at reached area

Other rolling: polyhedrons

Useable for rolling:

- Convex: can physically roll on faces
- Regular polyhedron as faces

All convex regular-faced polyhedrons!

Convex regular-faced poly

- 5 Platonic solids

Photo: MathsGear

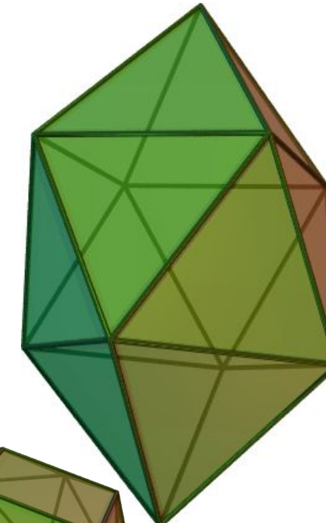
Convex regular-faced poly

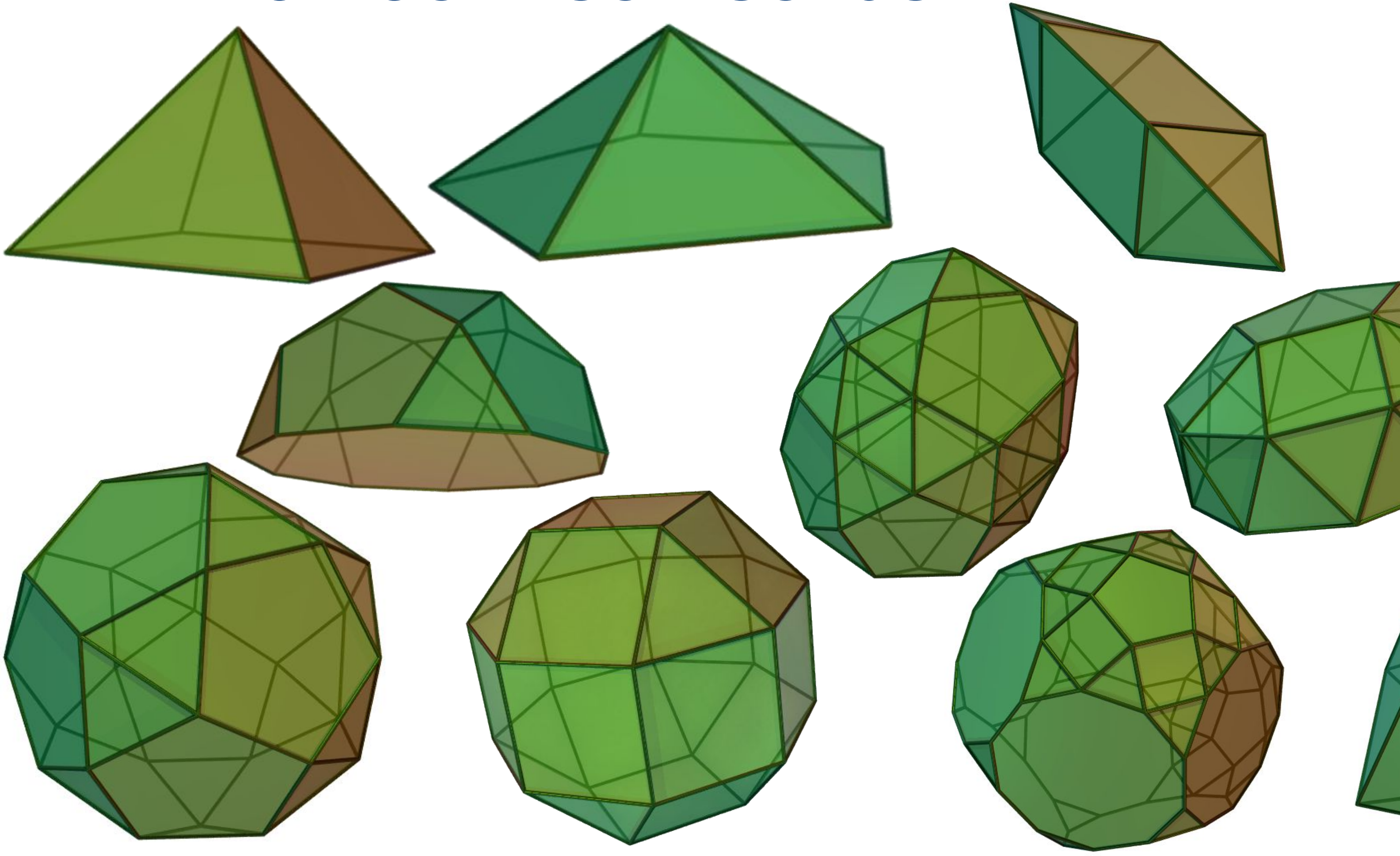
- 13 Archimedean solids

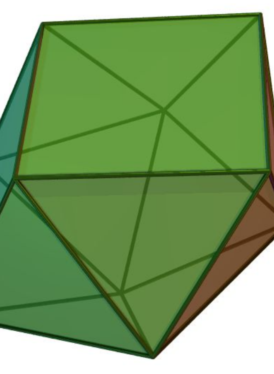
Convex regular-faced poly

- 92 Johnson solids

Renders via Wikipedia



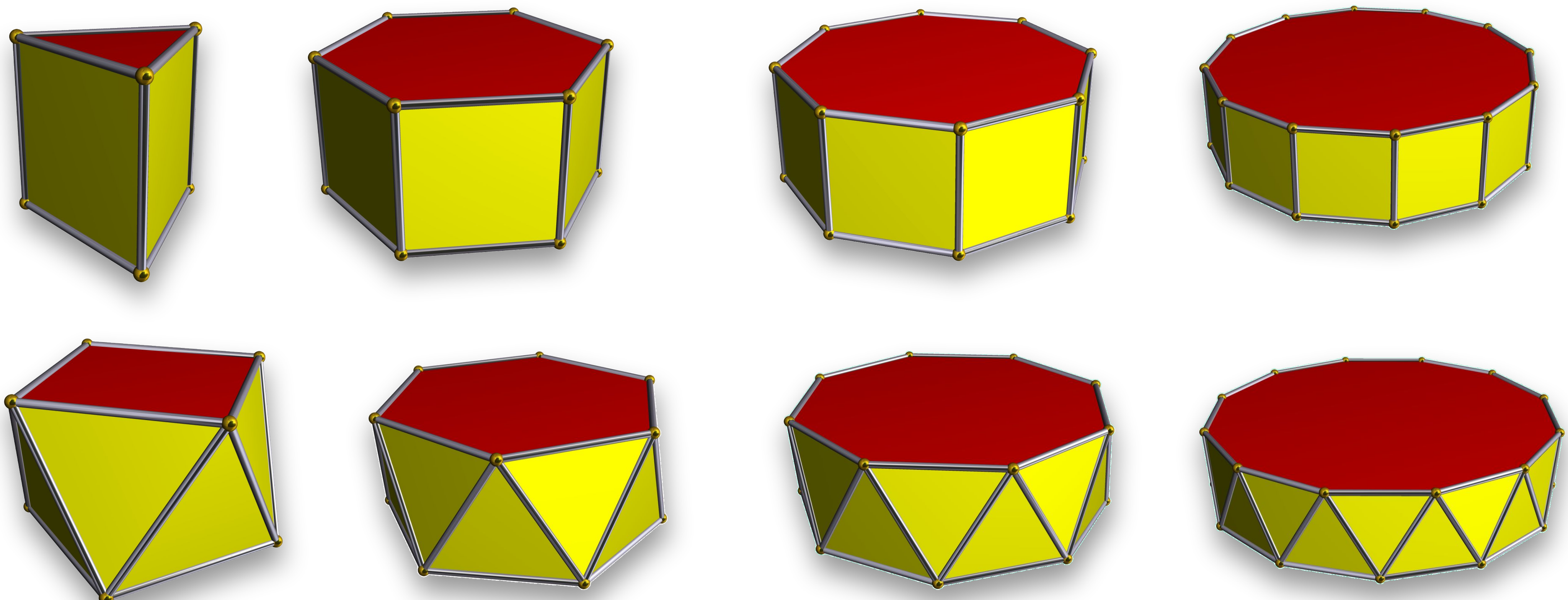




Convex regular-faced poly

- 4 Prisms: 3, 6, 8, 12
- 4 Antiprisms: 4, 6, 8, 12

Renders via Wikipedia



Convex regular-faced poly

- 5 Platonic solids
- 13 Archimedean solids + 2 chiral
- 92 Johnson solids + 5 chiral
- 4 Prisms
- 4 Antiprisms

125 polyhedron nets to check!

Other rolling: tilings

Useable for rolling:

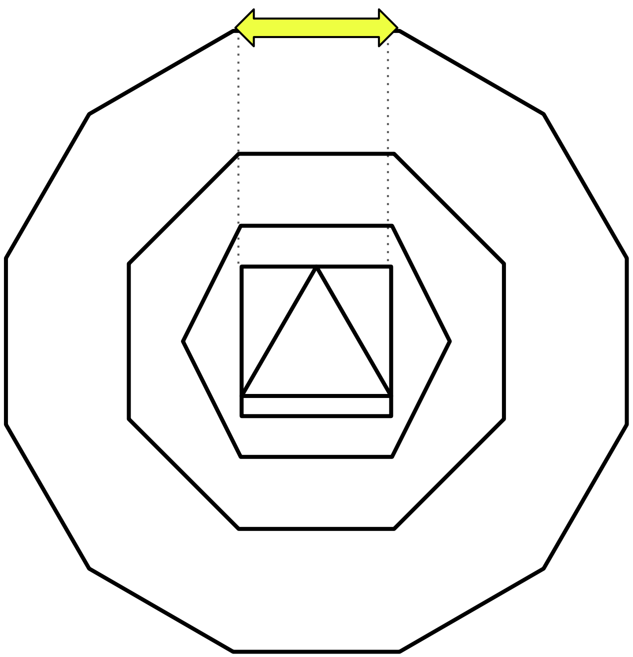
- Regular polygons as tiles
- Tiles the 2D plane
- Some form of description

→ k-uniform tessellations

Tessellations polygons

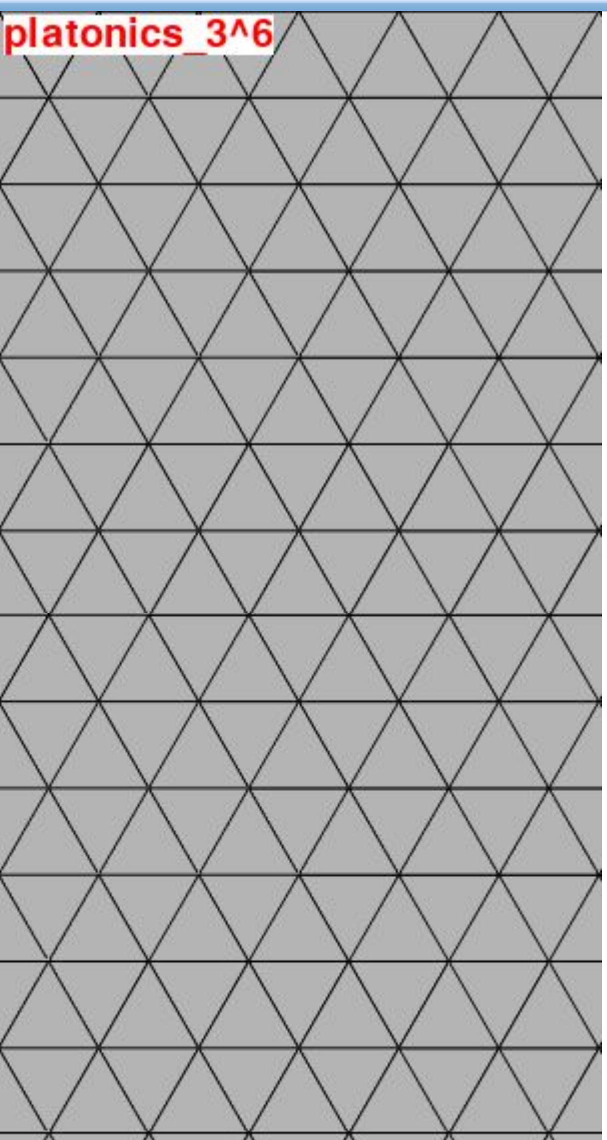
Useable n-gons in the 2D plane:

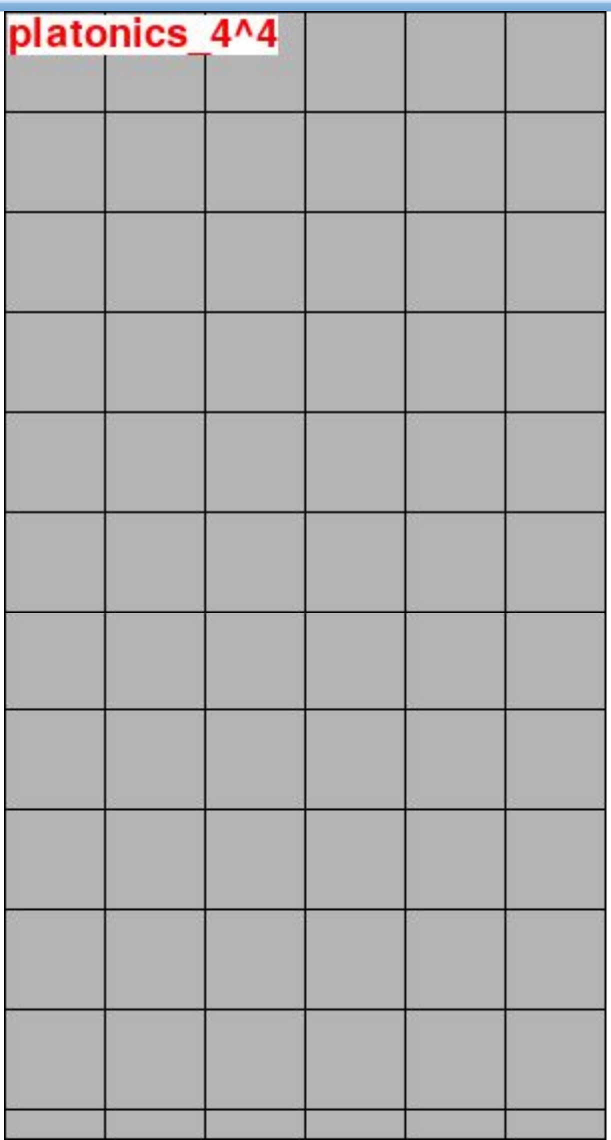
- Triangles
- Squares
- Hexagons
- Octagons
- Dodecagons

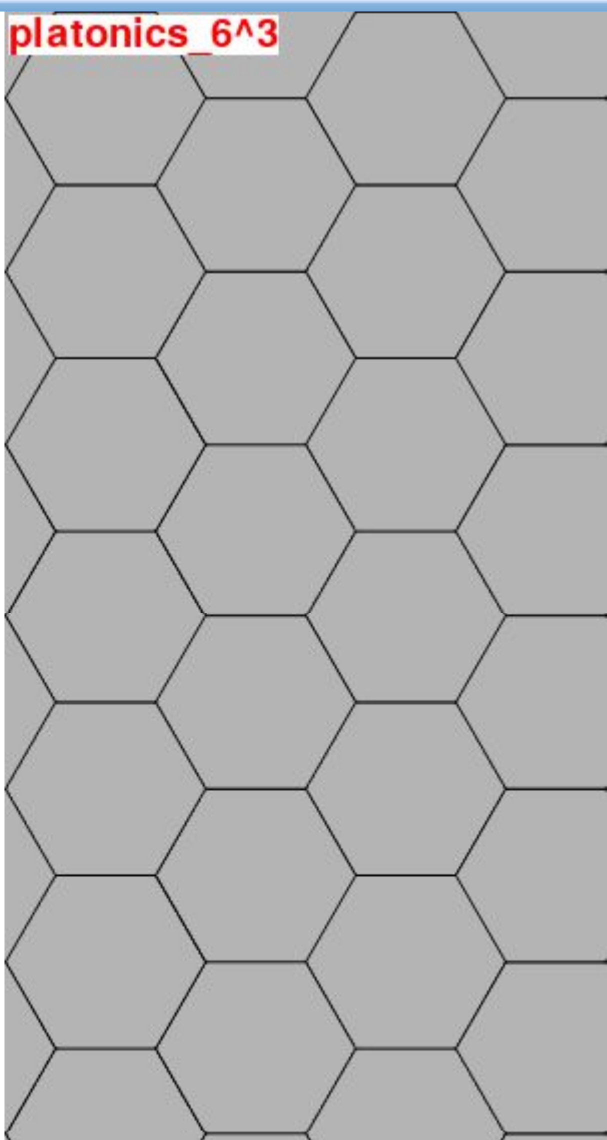


Angles that can sum to 360°

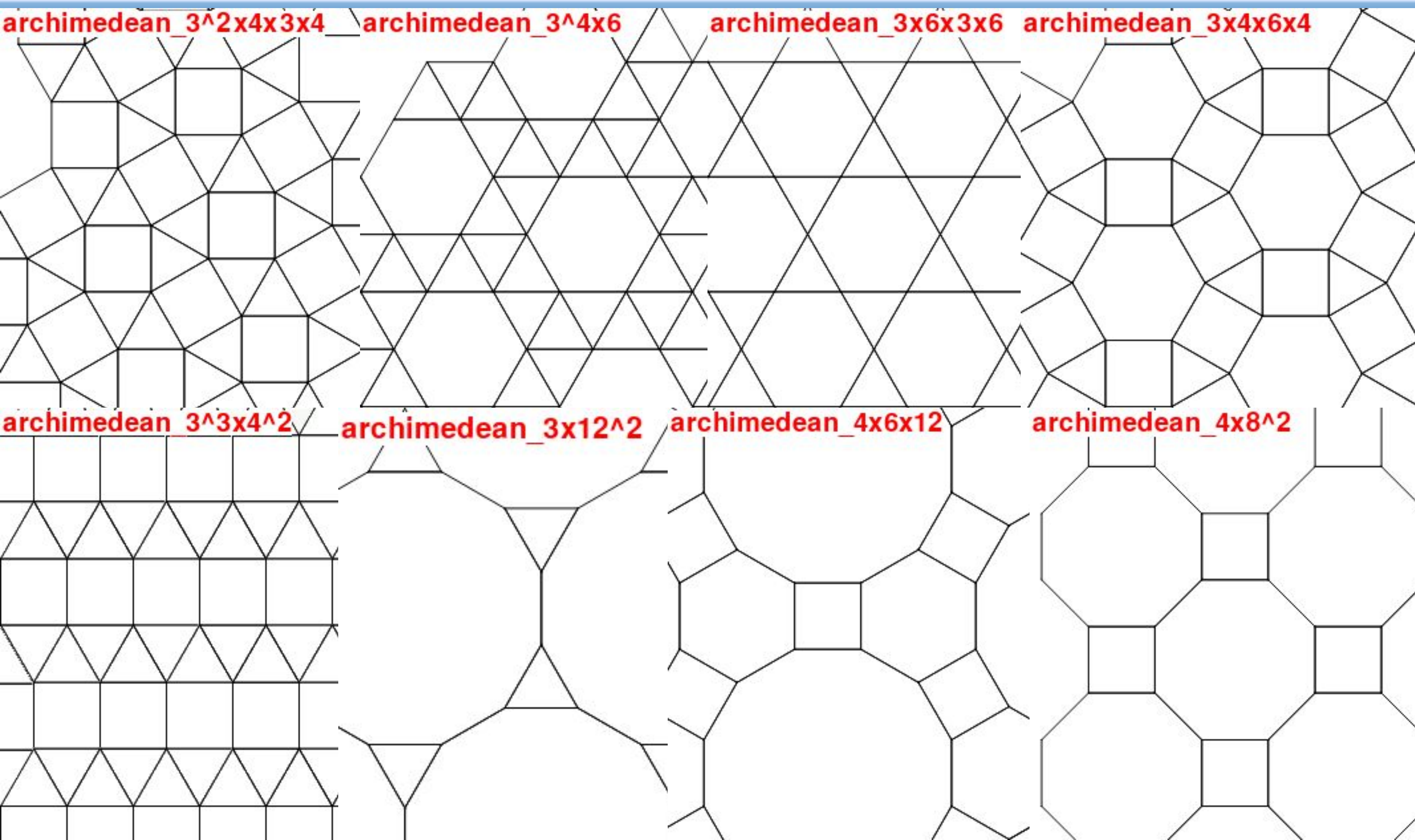
Other rolling: regular tilings





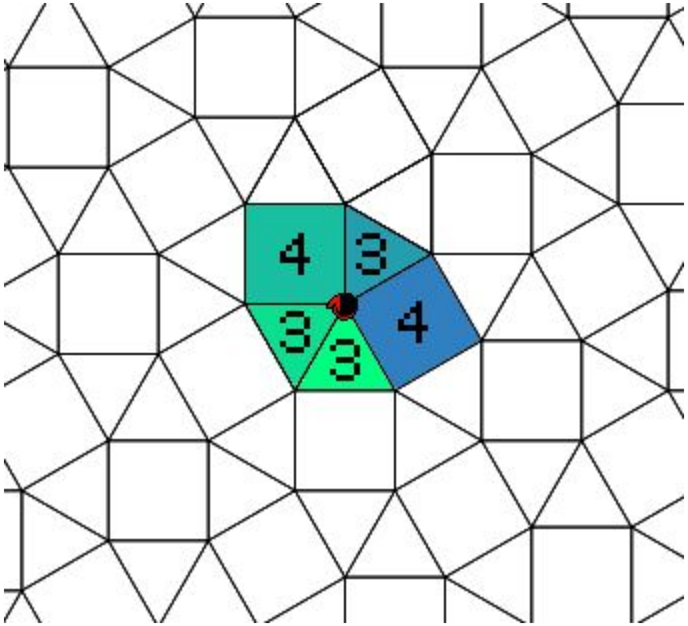


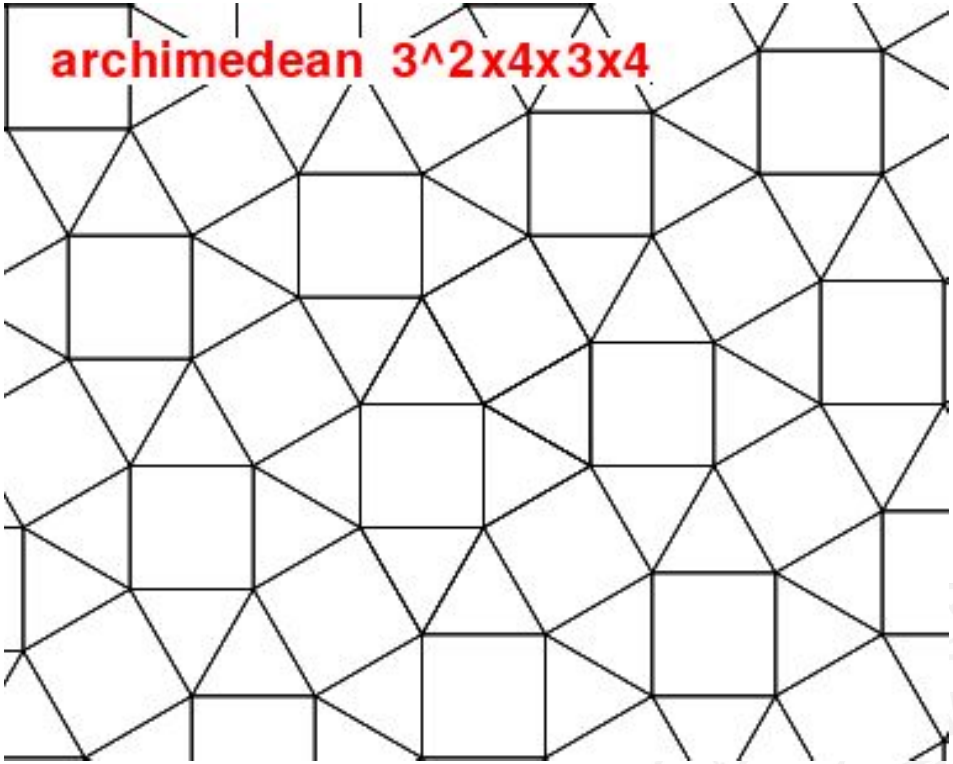
Other rolling: mixed tilings



Tiling description: Vertex

Vertex type = polygons next to vertex

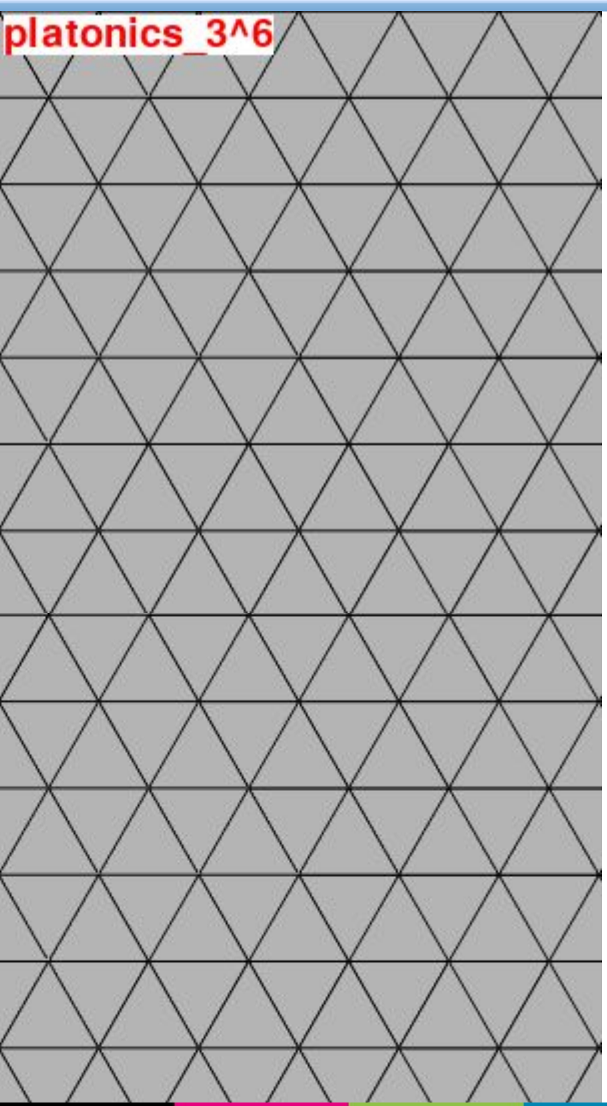


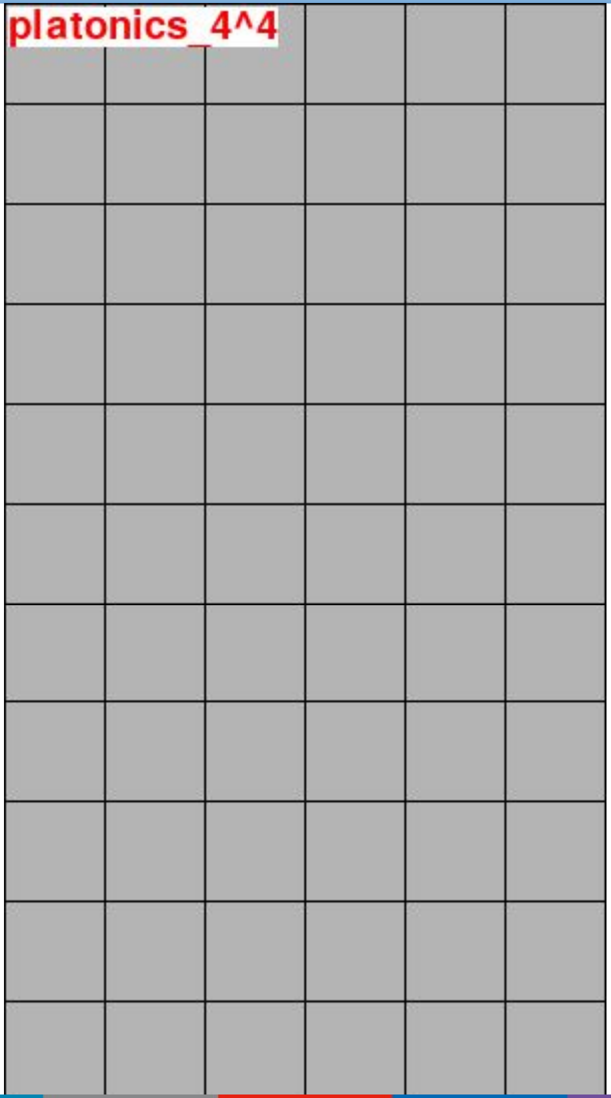


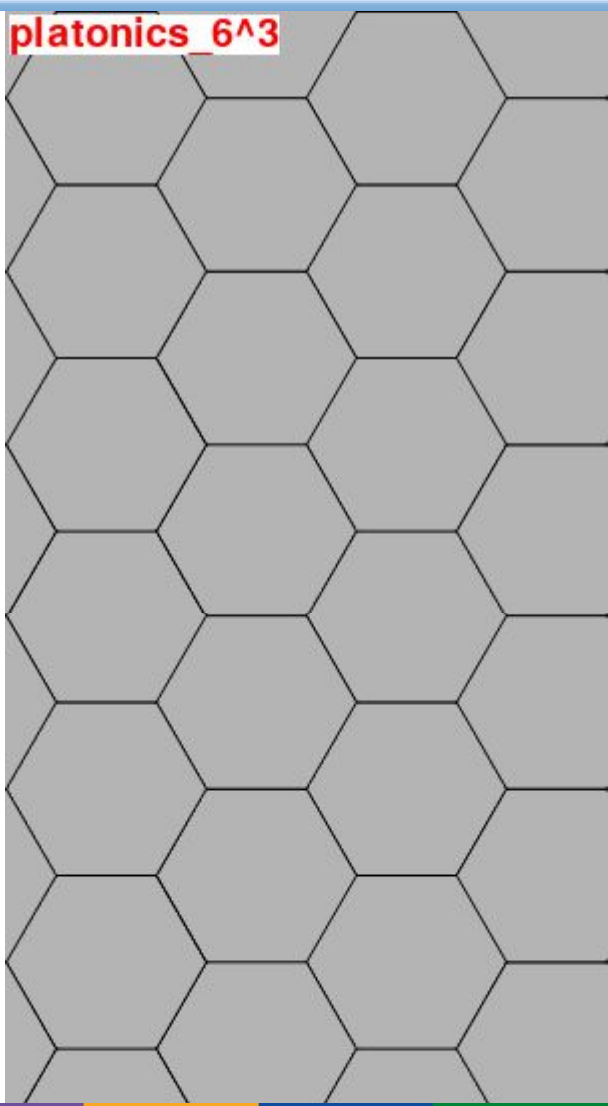
One vertex type:

3,3,4,3,4 repeated → tiling $3^2.4.3.4$

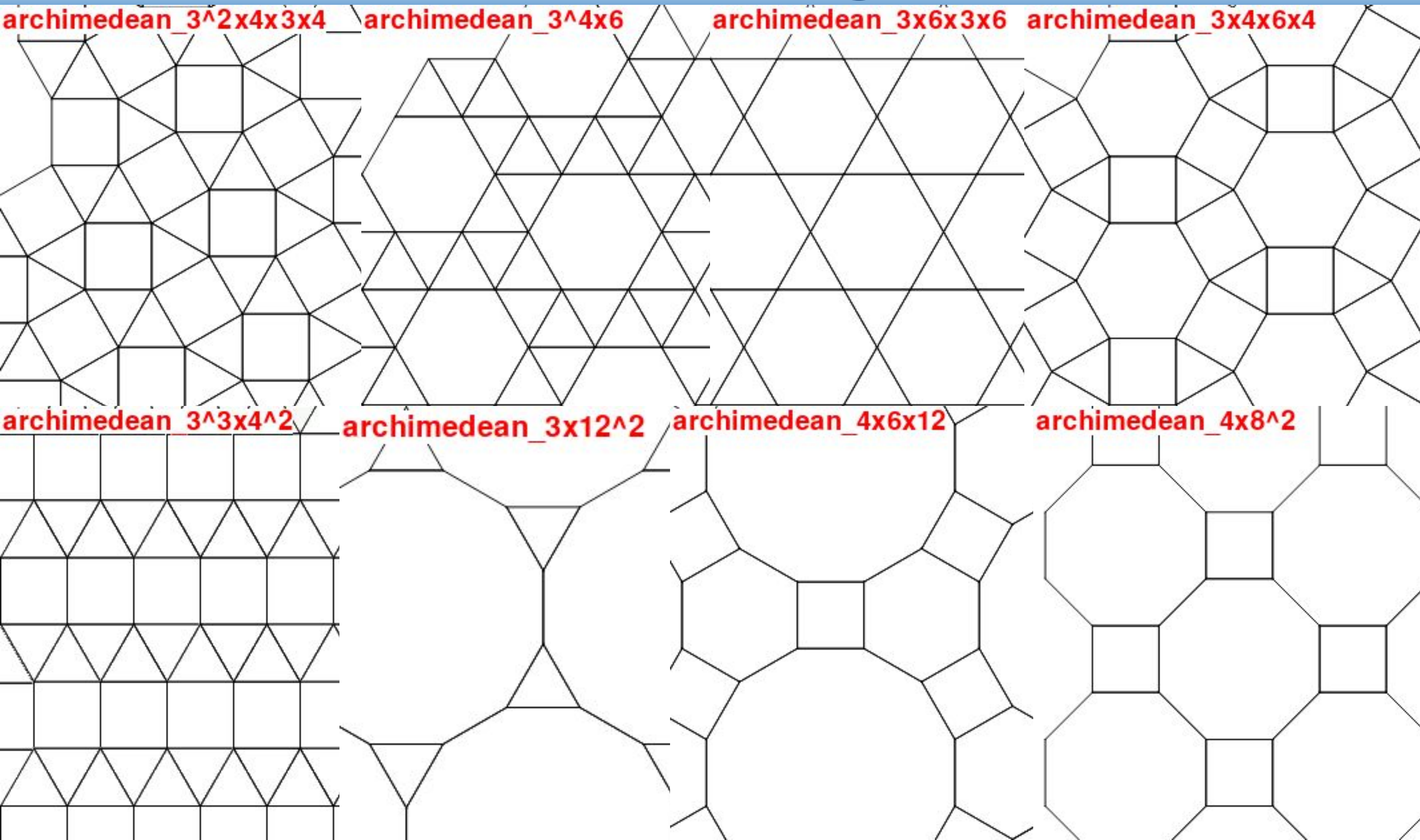
1-Uniform tilings: same polygon
3 Platonics tilings





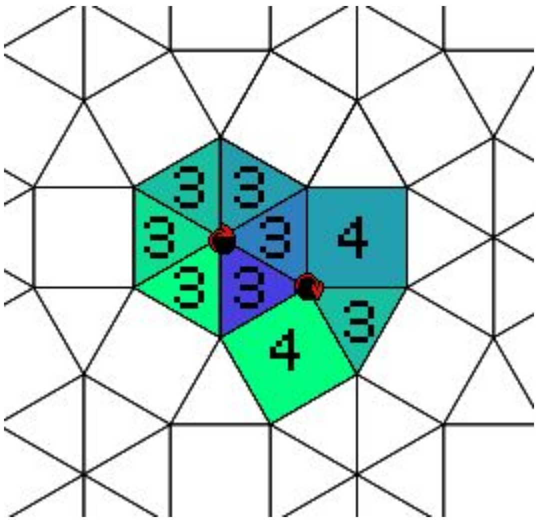


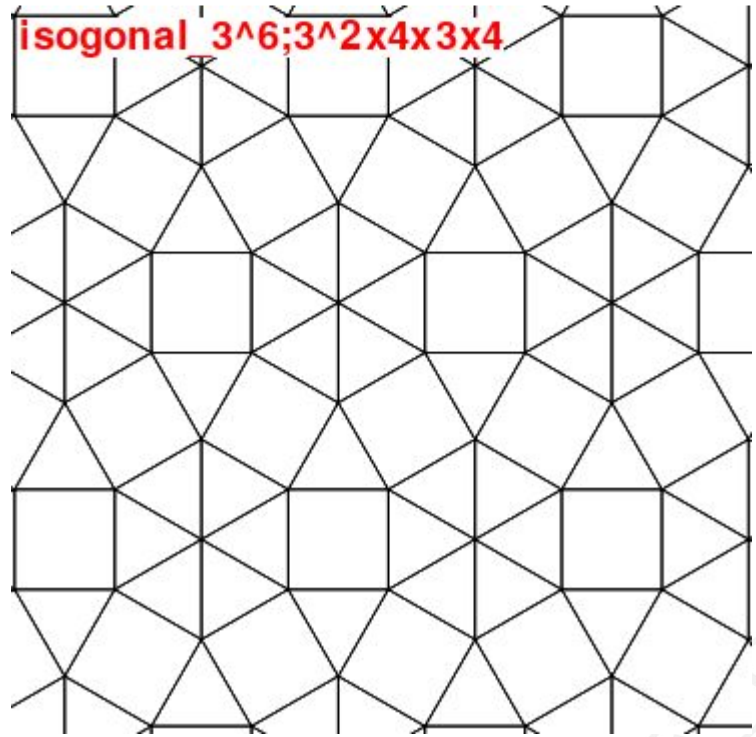
1-Uniform tilings: mixed polygons
8 Archimedeans tilings



2-Uniform Tessellations

Defined by **2** vertice types repeated

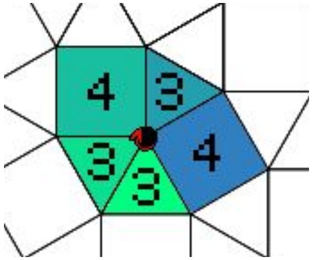


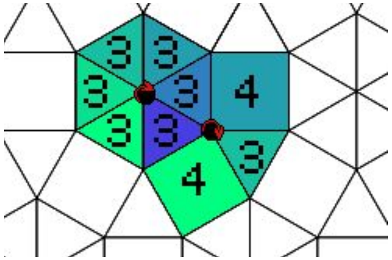


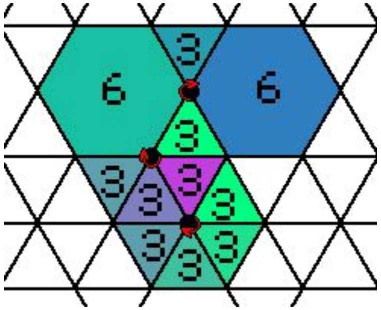
- 3,3,3,3,3,3
- 3,3,4,3,4 → tiling (3^6 ; 3^2.4.3.4)

N-Uniform Tessellations

- 1 vertex type:
  - 1-uniform (11 tilings)
- 2 vertex types:
  - 2-uniform (20 tilings)
- 3 vertex types:
  - 3-uniform (61 tilings)
- etc.







N-Uniform Tessellations

In my set:

- 1-uniform (11)
- 2-uniform (20)
- 3-uniform vertex-homogenous (39)

Only 70 tessellations tested

N-Uniform Tessellations (update)

In my set:

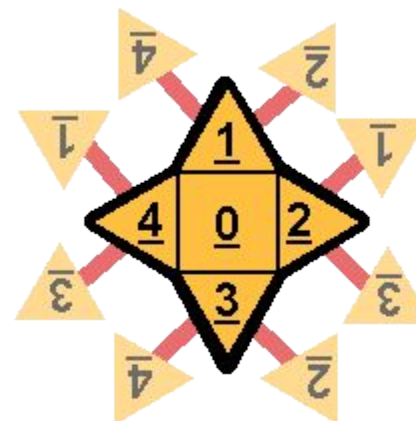
- 1-uniform (11)
- 2-uniform (20)
- 3-uniform (61)
- 4-uniform (39)

Only 131 tessellations tested

Rolling on a tessellation

Dual graph of the polyhedron

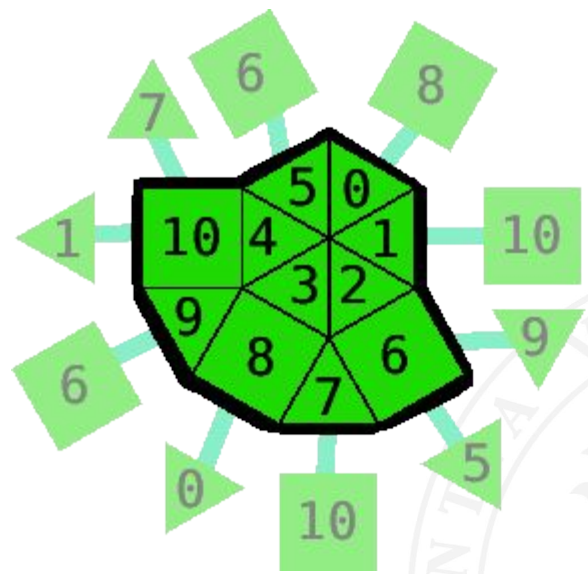
- Net of poly faces
- With orientation



Dual graph of the tiling

- Supertile with subtiles
- Parallelogon

(translations only)



Rolling on a tessellation

Rolling is:

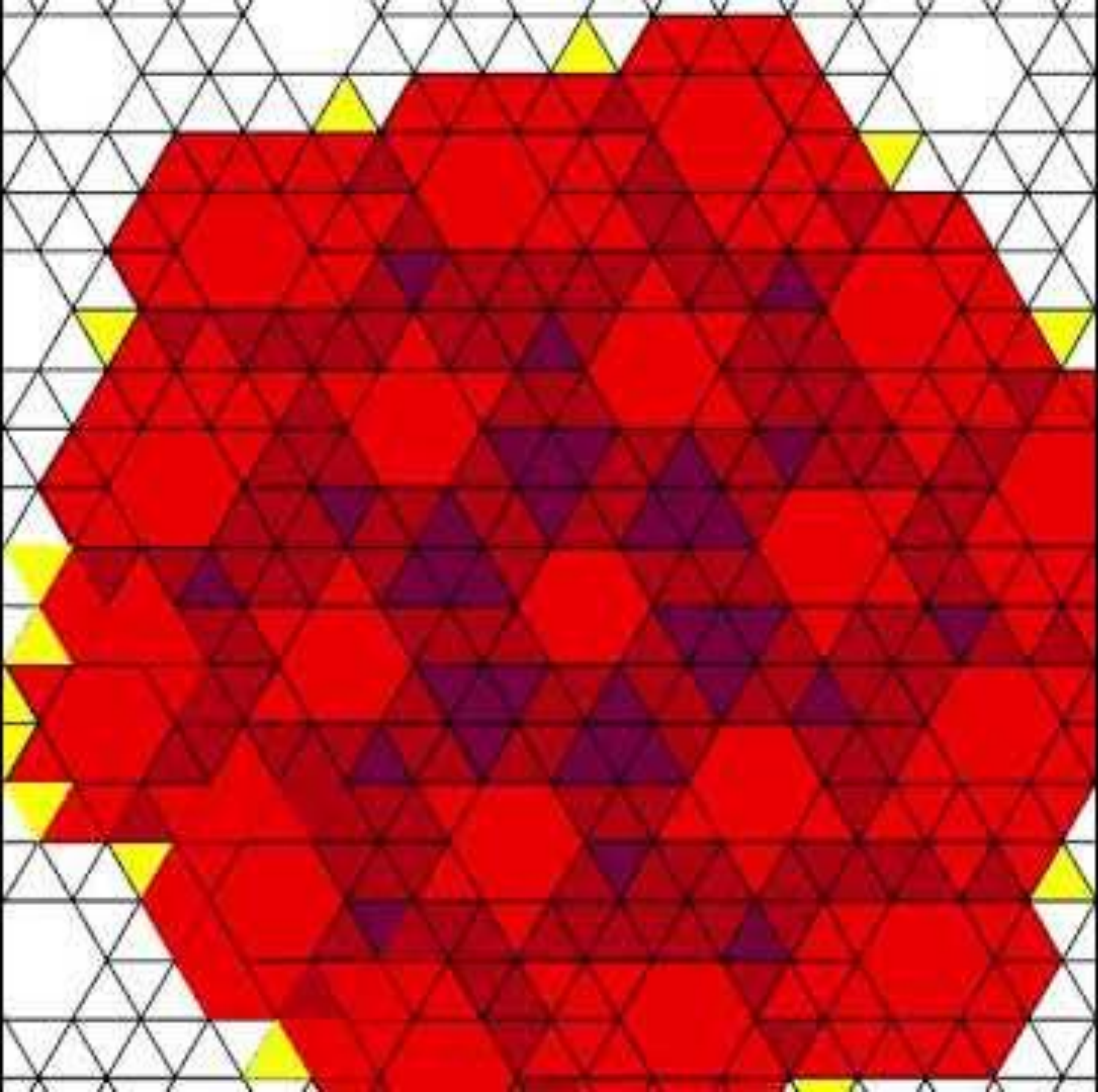
- Exploring the dual graph of
  - the polyhedron (net)
  - the tiling (tiles)
- while keeping track of the orientation
- and plane coordinates
- and matching the face/tile shape

Rolling on a tessellation

Rolling area = all reachable tiles

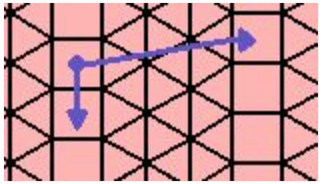
See video: we have different patterns!

All the plane, a band, a bounded area, of the plane but with holes.

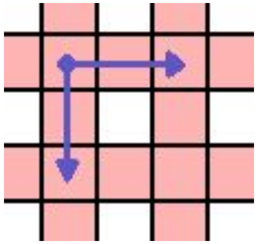


Different rolling patterns

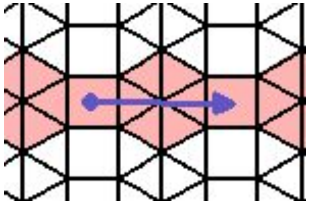
- Plane rollers
  - 2 symmetry vectors



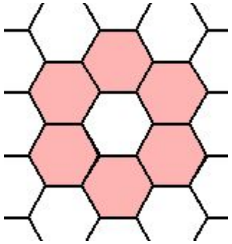
- Hollow plane rollers
  - 2 symmetry vectors
  - holes



- Band rollers
  - 1 symmetry vector



- Bounded area rollers
  - no symmetry vector



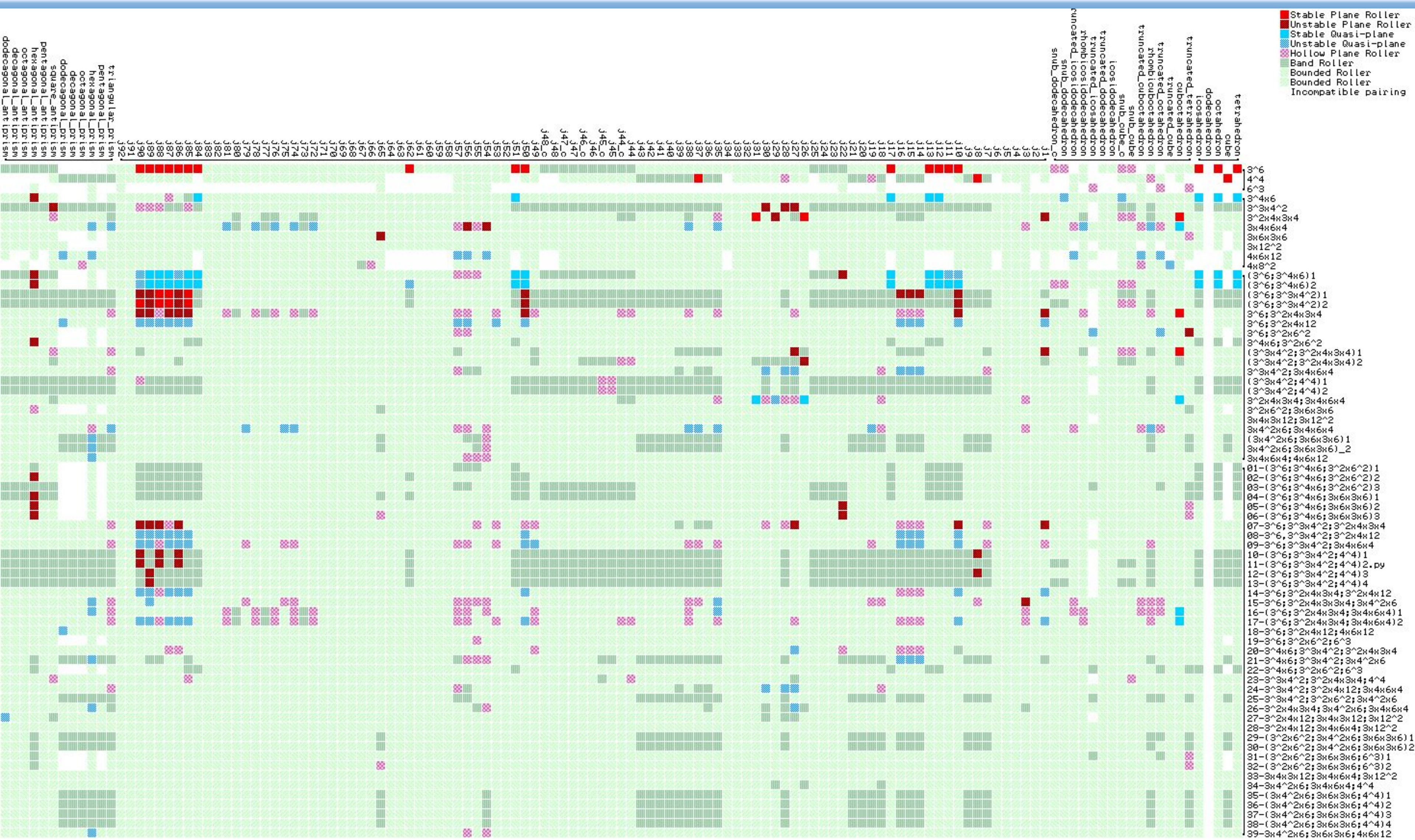
Determining patterns

Written a proof to computationally test a

- Polyhedron
- Tiling
- Starting position and orientation

And determine its biggest rolling area

Rolling pattern results



Results (updated)

125 polyhedrons & 131 tessellations

- 145 Plane rollers (44 stable)
- 588 Hollow Plane rollers (284 quasi)
- 2623 Band rollers
- 7362 Bounded rollers
- 581 incompatible pairs

Note: Stability

In some pairings, depending on the orientation, face, and tile where you start, you could roll the plane or get stuck.

Starting tiles that guarantee that you can roll the plane are called *stable tiles*.

A pair that is stable everywhere is a *stable roller*.

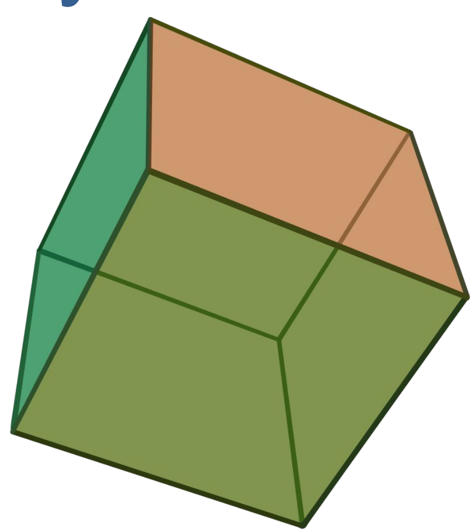
Desirable traits for rollers

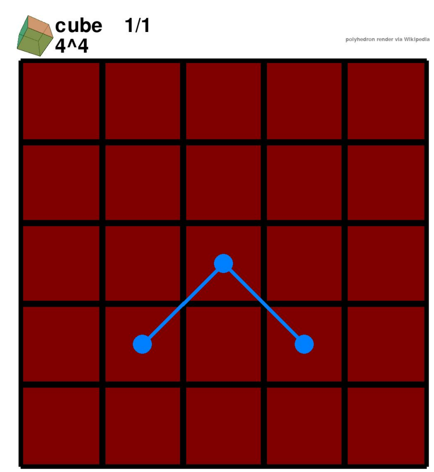
- You can start anywhere: stability
- Can roll everywhere: plane roller
  - with every face: face-complete
  - with every orientation: orientation-complete
- **Few faces?**

Desirable traits for rollers

Cube on Square Grid:

- Can start anywhere: stable
- Can roll everywhere: plane roller
    - with every face: face-complete
- 6 faces



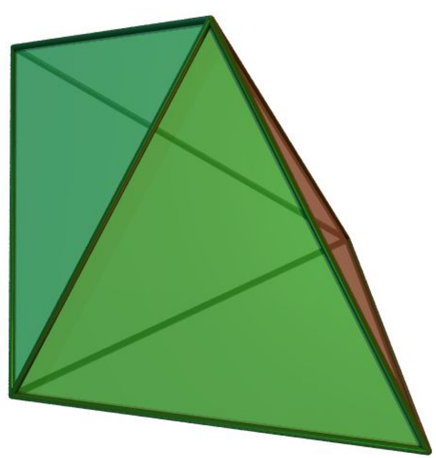


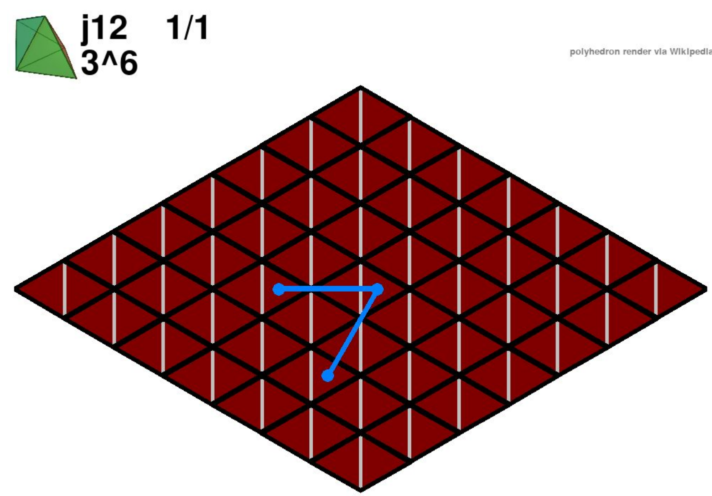
Desirable traits for rollers

J12 (triangular bipyramid) on Triangle Grid:

- Can start anywhere: stable
- Can roll everywhere: plane roller
  - with every face: face-complete
- 6 faces

(2 tetrahedrons)



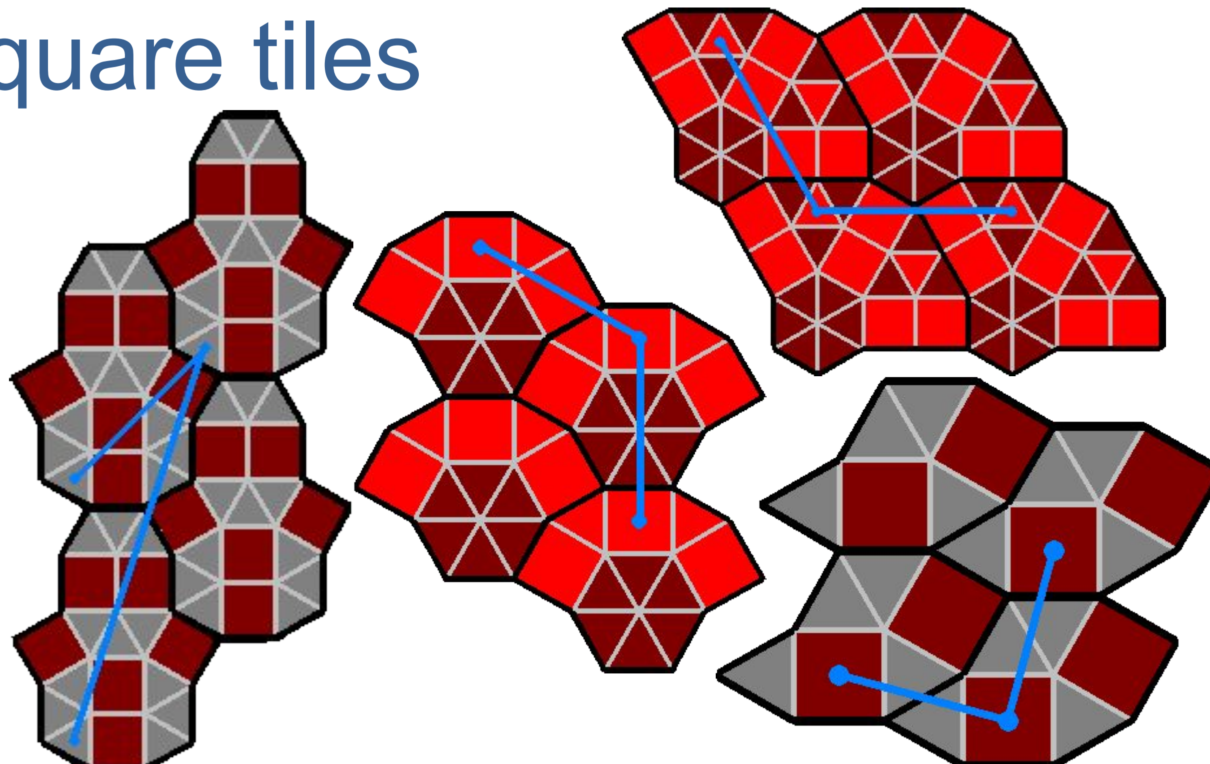


polyhedron render via Wikipedia

Desirable traits for rollers

J1 (square pyramid) on $(3^6;3^2 4.3.4)$, $3^2 4.3.4$, $(3^3 4^2;3^2 4.3.4)_1$, $(3^6;3^3 4^2;3^2 4.3.4)$:

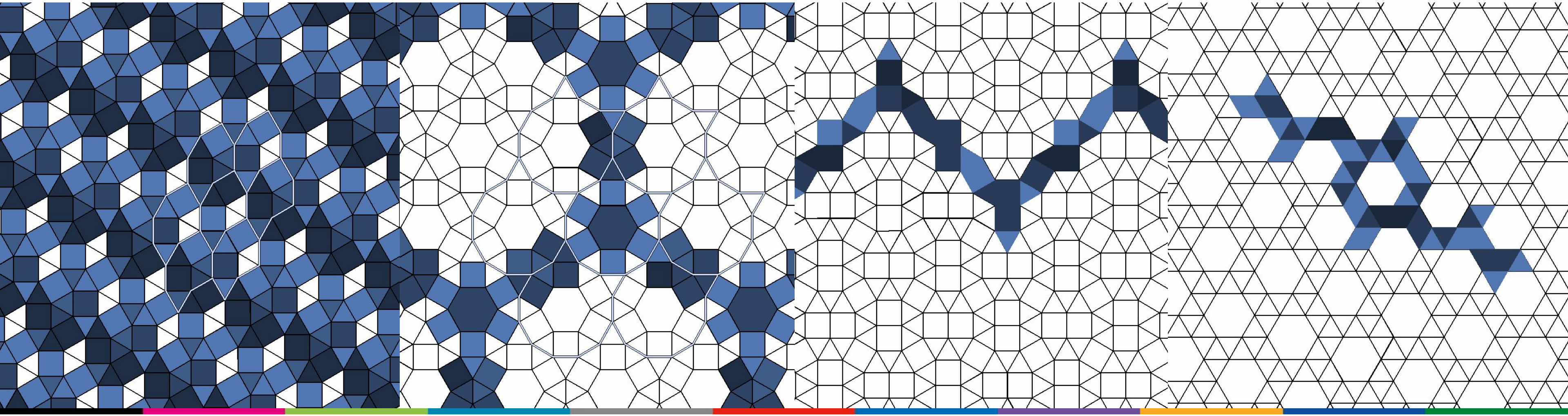
- Stable on square tiles
- Plane roller
- 5 faces



Desirable traits for rollers

All the other rolling pairs are useable

- Quasi-plane rollers: avoid incompatible
- Hollow plane rollers: patterns
- Band and Area rollers: in action games



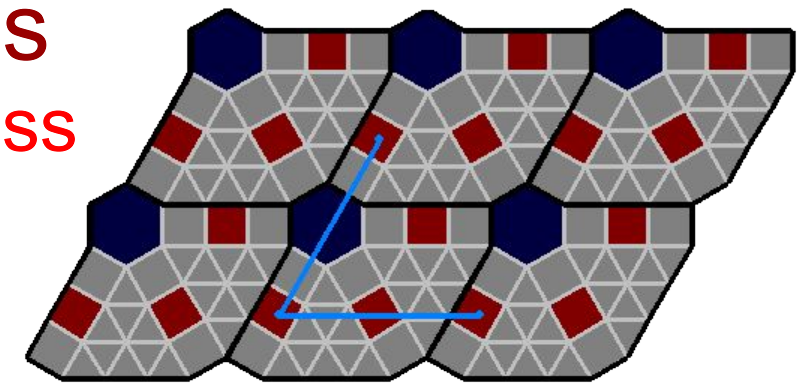
Recapitulative images

j86 1/88

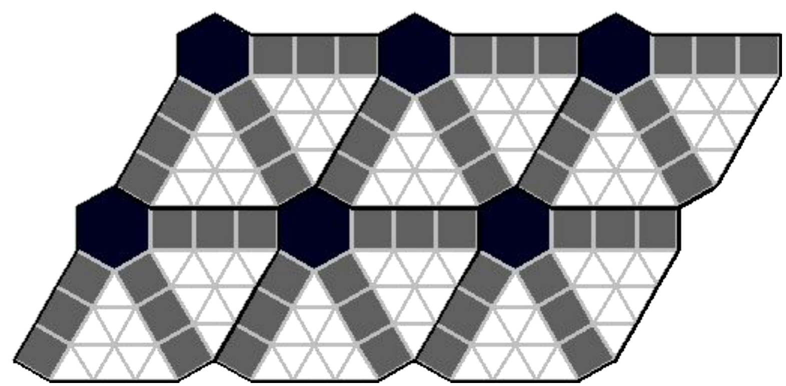
polyhedron render via Wikipedia

09-3^6;3^3x4^2;3x4x6x4

- Reached tiles
- Face-completeness
- Orientation-completeness



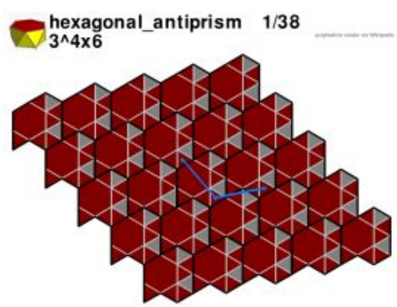
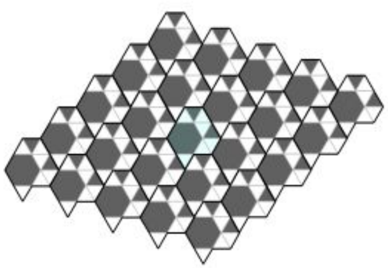
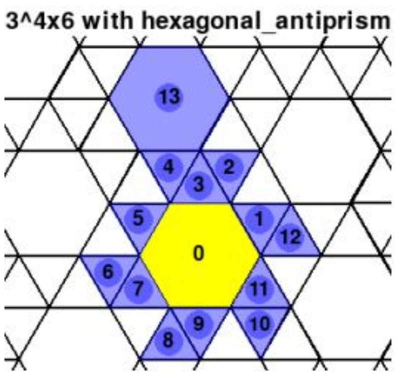
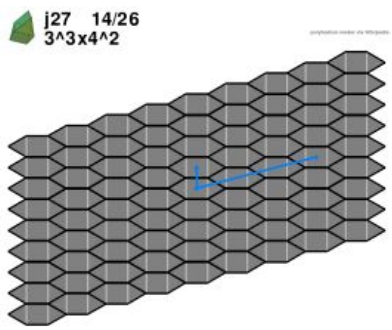
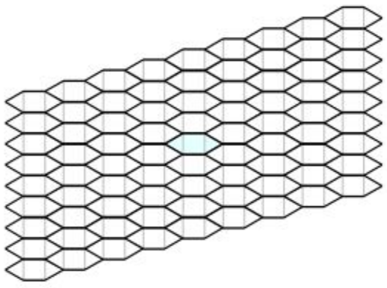
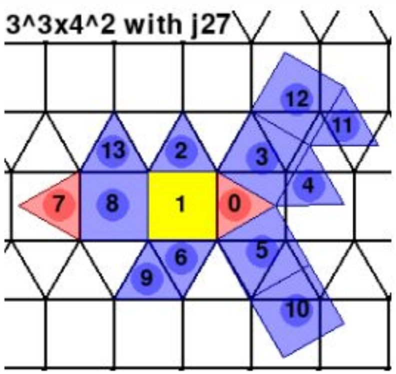
- Incompatible tiles



- Stable tiles (grey)
- Unstable (white)

Recapitulative images

Data available for all puzzle makers!

Roller Pair	Reachability	Stability	Faces
hexagonal antiprism 3^4x6			
j27 3^3x4^2			

https://drive.google.com/file/d/17RAs01OhV3DfONttQ8w_KTIjFKHSPUWY

Future research

There is more to be done!

- Hyperbolic plane tilings, aperiodic, radial (spirals? branches?)
- Generate tilings
- Non-regular, Concave polyhedron
- More data to extract for puzzles: path length, rotations

Conclusion

I hope this work will lead to more polyhedrons being used in puzzles!