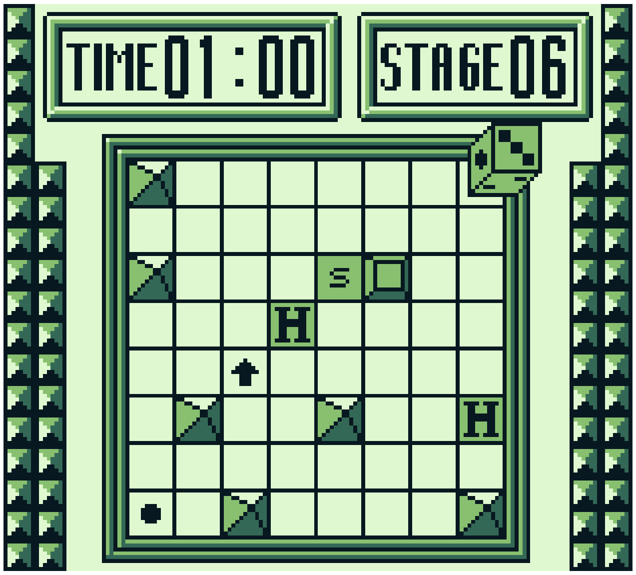
Puzzlemaker's reference for Rolling Polyhedron
Part 1: Plane rollers

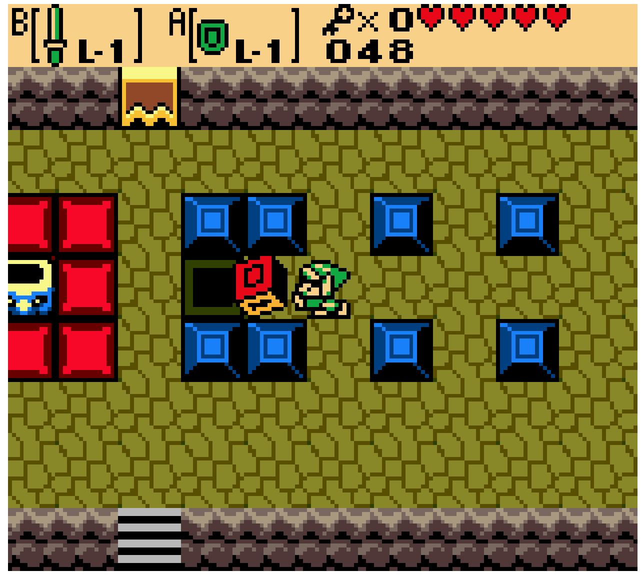
CCBY Akira BAES @akirabaes V1.0

Rolling puzzles often feature *cubes*, rarely other types of *dice*.

This document will list alternative shapes (**regular-faced convex polyhedrons**) that can be used on specific **n-uniform tilings** (tessellations using regular polygons as tiles). Such a pair is called a **Roller**.

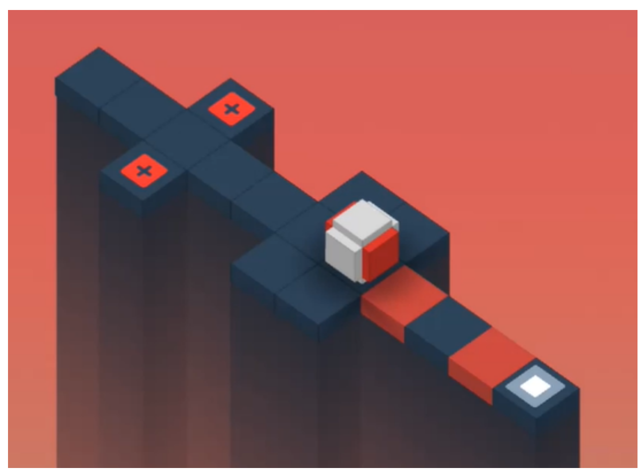


(a) *Korodice* (1990)



(b) *Zelda Oracle of Ages* (2001)

(c) *Devil Dice* (1998)



(d) *Rubek* (2016)

Figure 1: Examples of face-matching rolling cube puzzles

Polyhedrons list includes Platonic solids, Archimedean solids, n-Prisms and Antiprisms. Polyhedron with the format JXX are called Johnson solids. With chirals. Dataset: Polyhedron dual as python dictionaries.

Tilings are named by their vertex configuration orbits notation. So far, I used the first 90 tilings of this Catalog. Dataset: Tilings as python dictionaries.

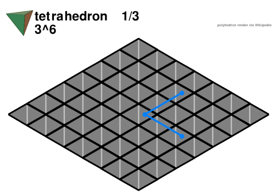
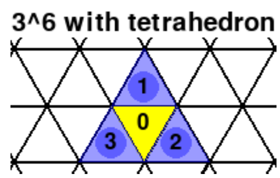
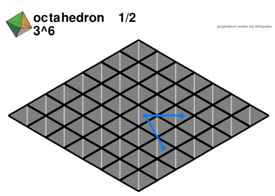
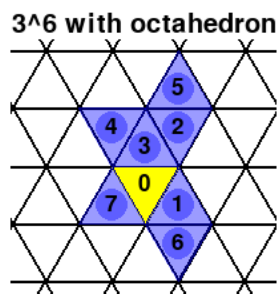
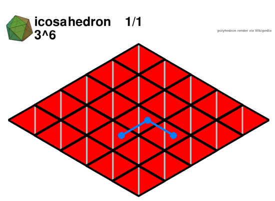
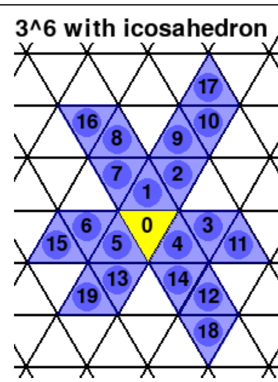
1 Stable plane rollers

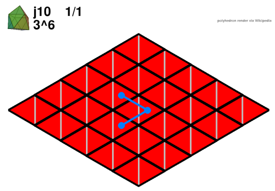
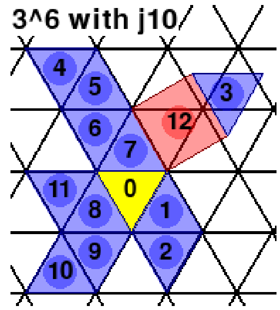
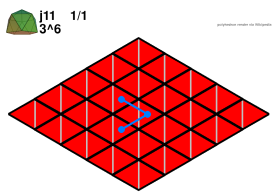
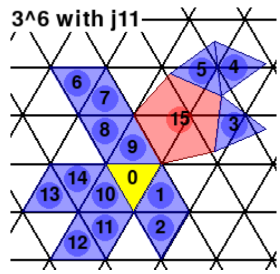
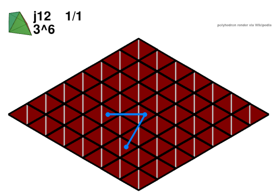
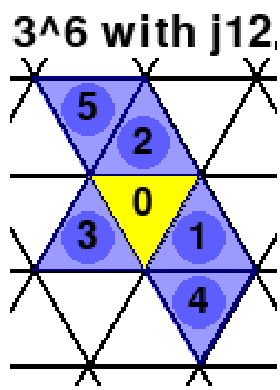
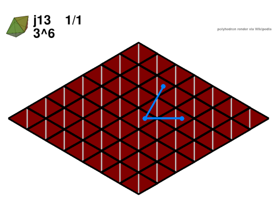
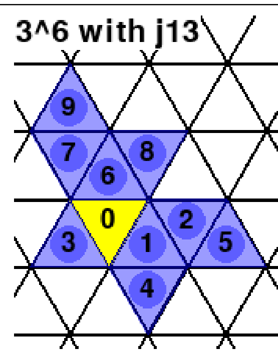
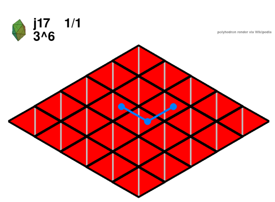
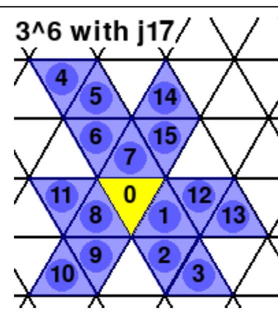
A plane roller is a polyhedron that covers the whole plane by rolling on a tiling. A stable plane roller is a plane roller for every starting position.

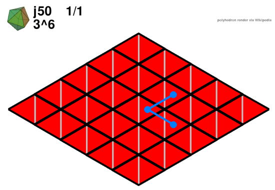
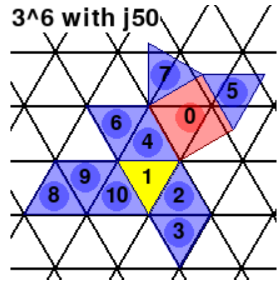
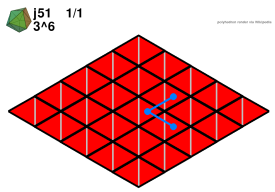
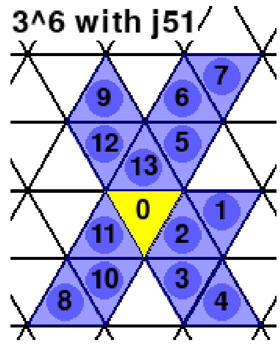
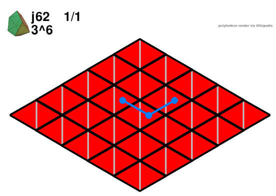
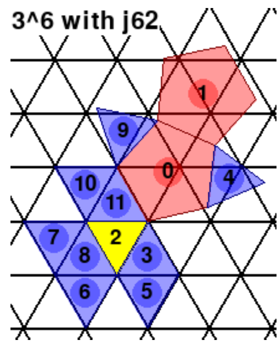
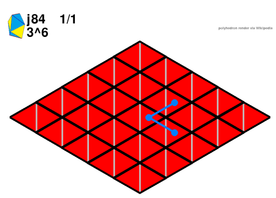
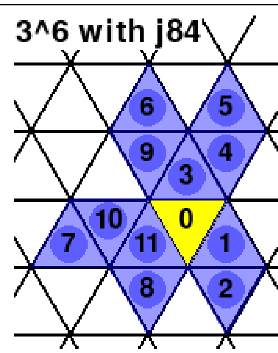
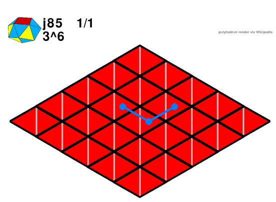
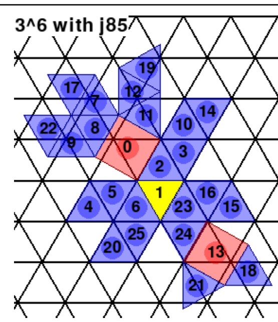
1.1 Legend

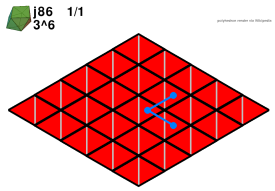
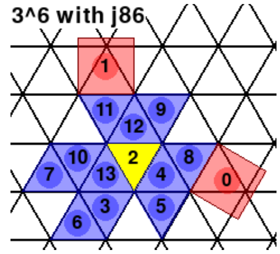
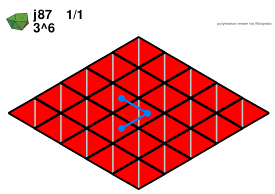
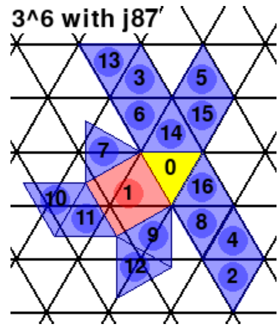
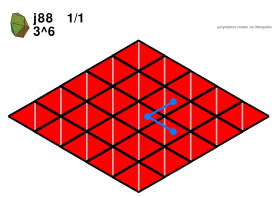
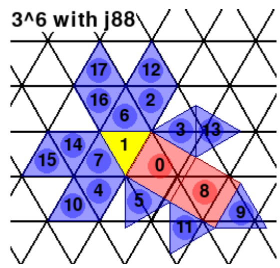
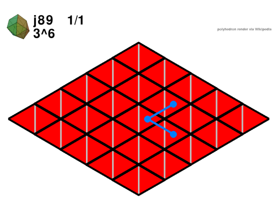
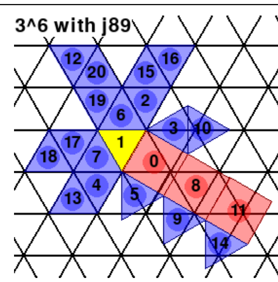
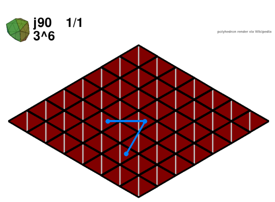
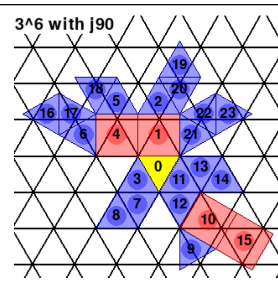
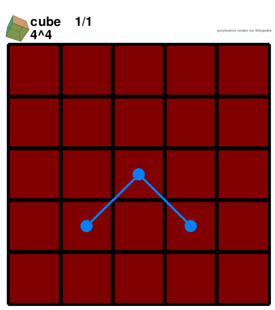
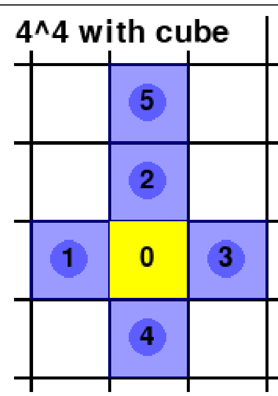
- Grey cells: reached areas
- Brown cells: areas reachable with every compatible faces from the polyhedron (ex. all triangles of the pyramid)
- Red cells: areas reachable with every compatible faces from the polyhedron in all orientations (ex. all triangles of the pyramid in all orientations)
- On the right: preview of the net of the polyhedron, with used faces (blue), and unused faces (red).

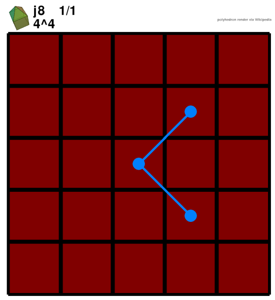
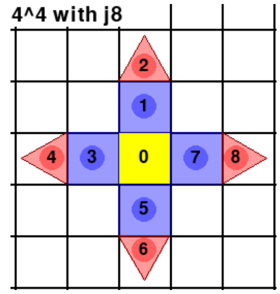
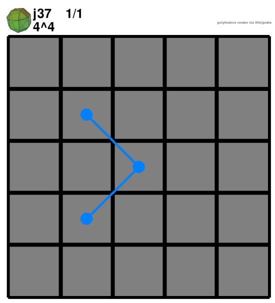
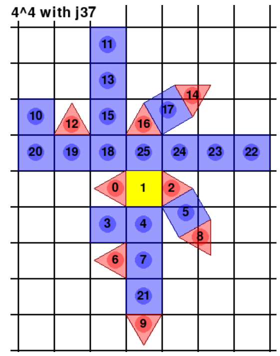
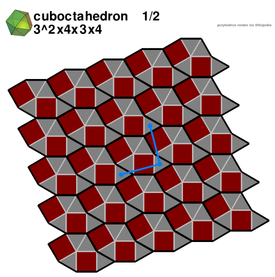
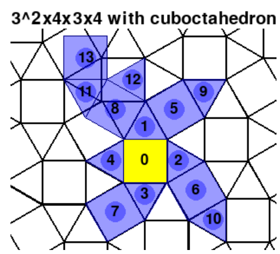
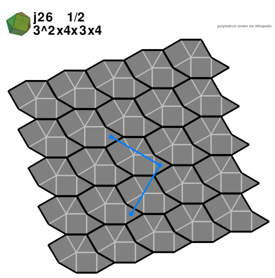
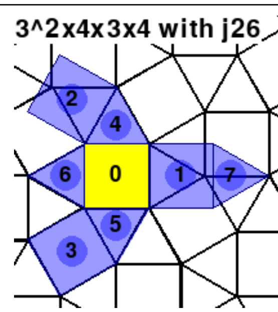
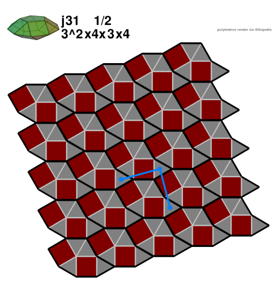
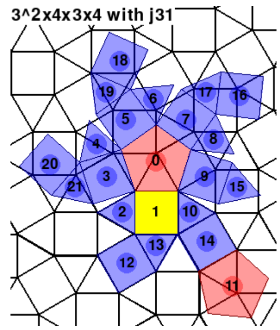
1.2 Table

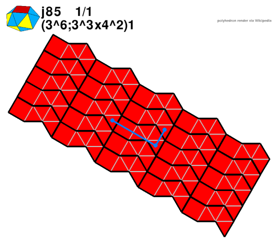
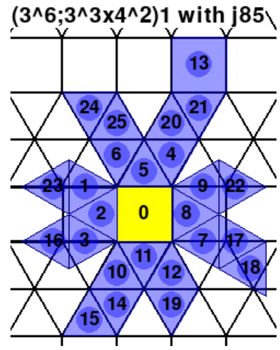
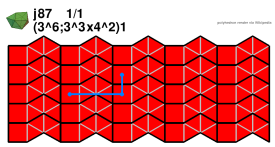
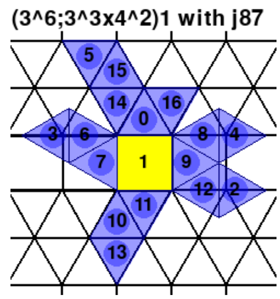
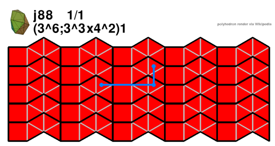
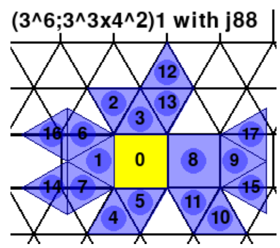
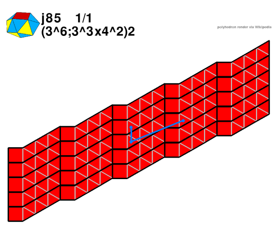
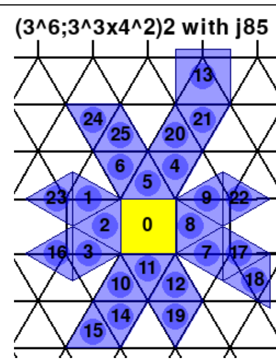
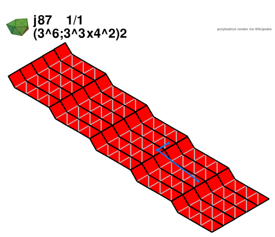
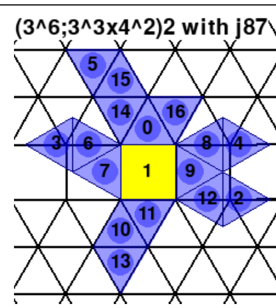
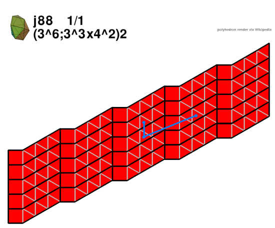
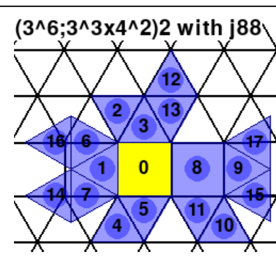
Roller Pair	Reachability	Faces
tetrahedron 3^6		
octahedron 3^6		
icosahedron 3^6		

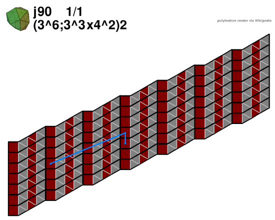
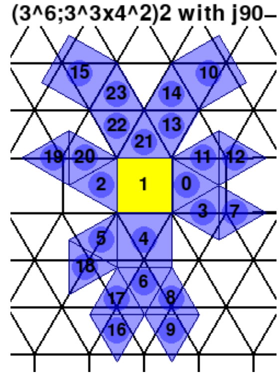
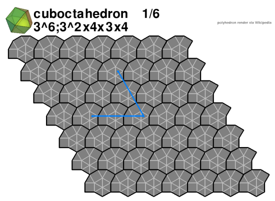
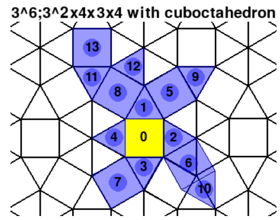
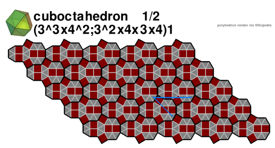
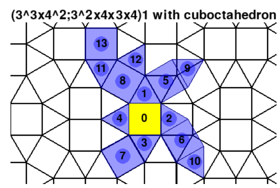
j10 3^6		
j11 3^6		
j12 3^6		
j13 3^6		
j17 3^6		

j50 3^6		
j51 3^6		
j62 3^6		
j84 3^6		
j85 3^6		

j86 3^6		
j87 3^6		
j88 3^6		
j89 3^6		
j90 3^6		
cube 4^4		

j8 4^4		
j37 4^4		
cuboctahedron $3^2x4x3x4$		
j26 $3^2x4x3x4$		
j31 $3^2x4x3x4$		

j85 $(3^6; 3^3x4^2)1$		
j87 $(3^6; 3^3x4^2)1$		
j88 $(3^6; 3^3x4^2)1$		
j85 $(3^6; 3^3x4^2)2$		
j87 $(3^6; 3^3x4^2)2$		
j88 $(3^6; 3^3x4^2)2$		

j90 $(3^6; 3^3x4^2)2$		
cuboctahedron $3^6; 3^2x4x3x4$		
cuboctahedron $(3^3x4^2; 3^2x4x3x4)1$		

1.3 Highlights

On the square tiling: The cube can roll on the square tiling with all faces. However, each face can only reach every tile in two given orientations. In this context J8 behaves like a cube with a missing face and can also roll the tiling. J37 might be more adapted for action-games.

Many shapes can roll the triangular tiling. One shape to highlight: J12 because it is very similar to the cube: face-complete rolling, and only 6 faces.

Tetrahedron and Octahedron do not have face-completeness on the tiling so they are not suited for complex face-matching puzzles, which might explain their lack of use despite being an easy to guess plane roller.

Tilings with rows of squares interrupted by triangles have a lot of potential polyhedron pairings.

2 Unstable plane rollers

Unstable rollers have some tiles that are not guaranteed to produce a plane rolling pattern. Starting on some tiles with some face on some orientation might lead to the shape getting stuck in a smaller pattern. It is recommended to start on a stable tile (second column) or in a known starting position (right column).

2.1 Legend

Rolling area (left)

- Grey cells: reached areas
- Brown cells: areas reached with every compatible faces from the polyhedron
- Red cells: areas reached with every compatible faces from the polyhedron in all orientations

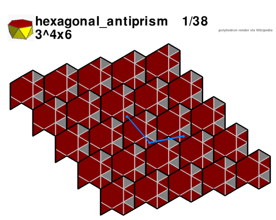
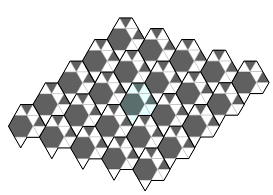
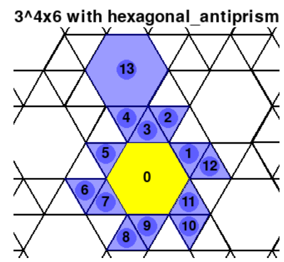
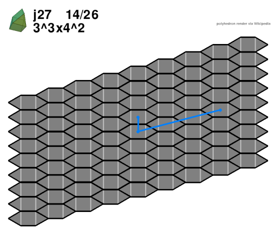
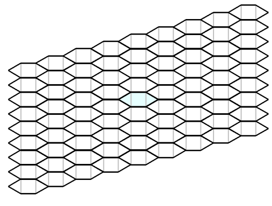
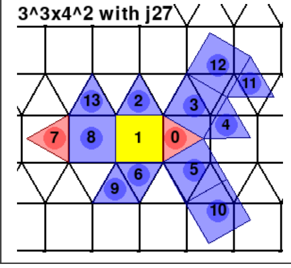
Stability graph (middle)

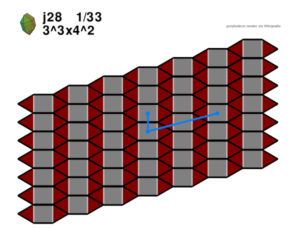
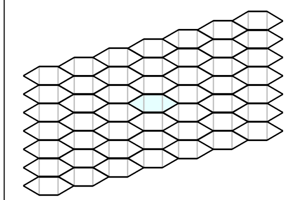
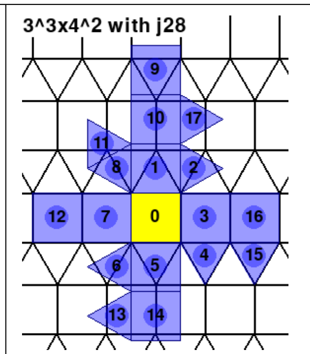
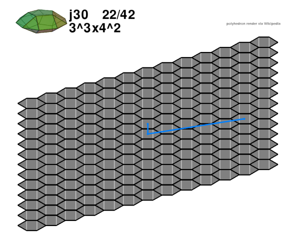
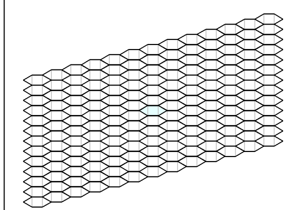
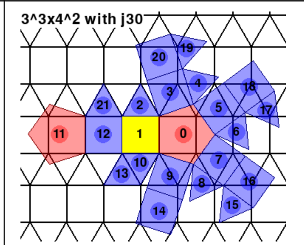
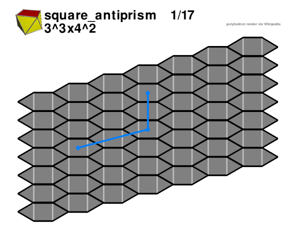
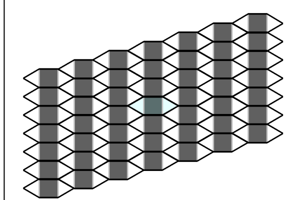
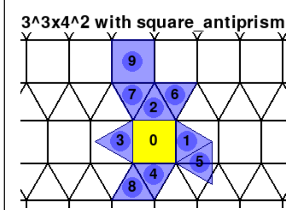
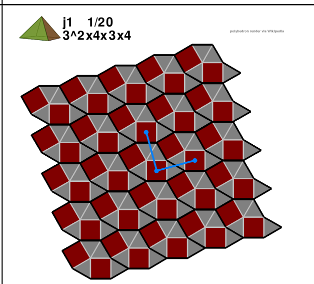
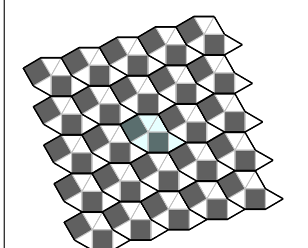
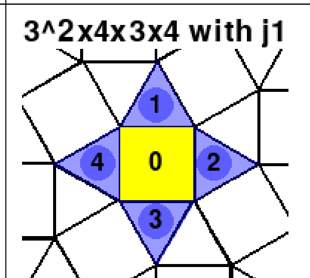
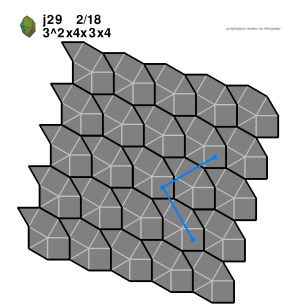
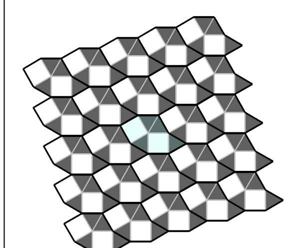
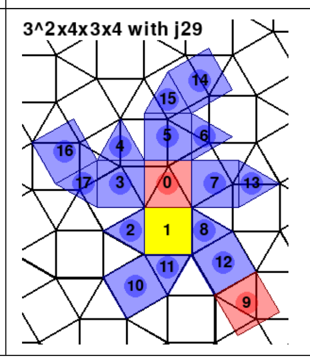
- Grey cells: stable cells: the structure of the rolling graph does not change depending on the face/orientation that starts on this area
- White cells: unstable cells, might lead to the roller getting stuck in an area/band

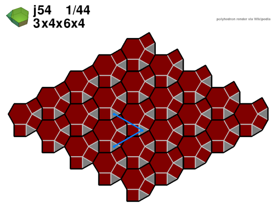
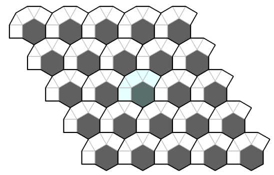
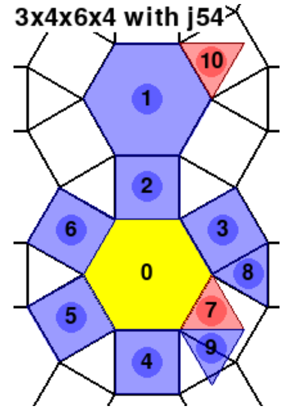
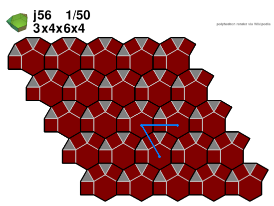
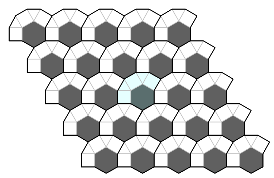
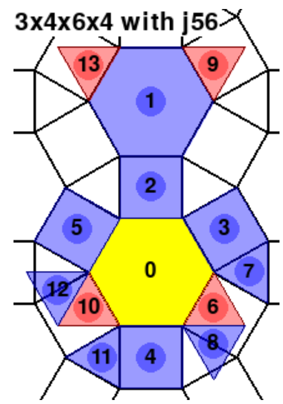
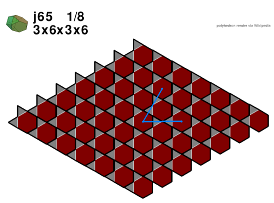
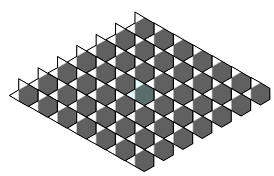
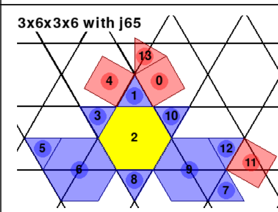
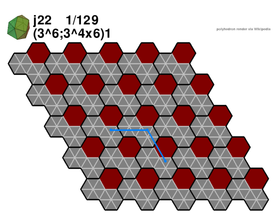
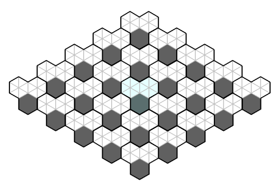
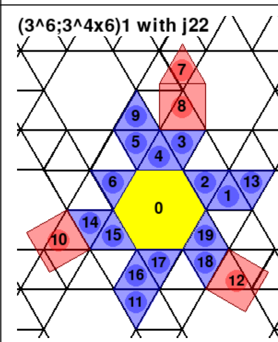
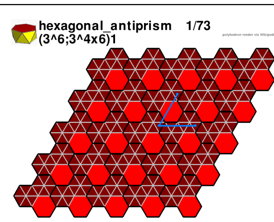
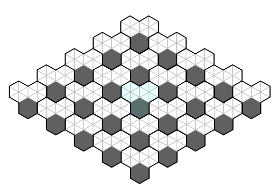
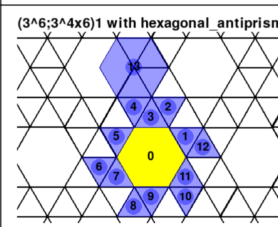
Starting position (right)

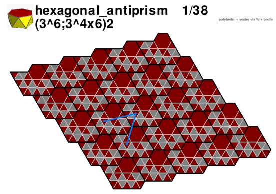
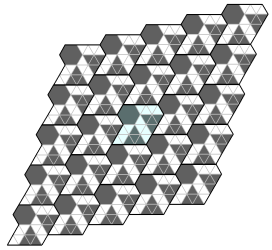
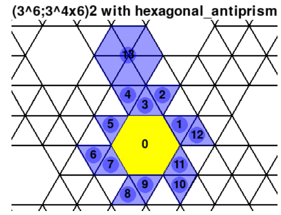
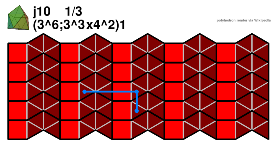
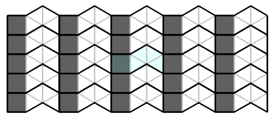
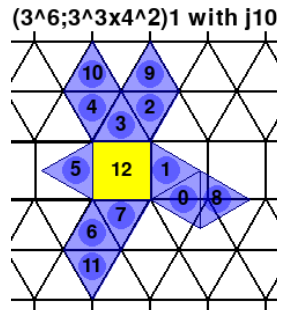
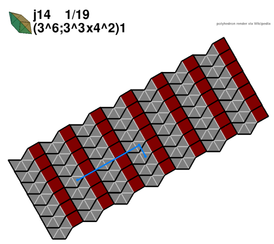
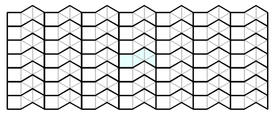
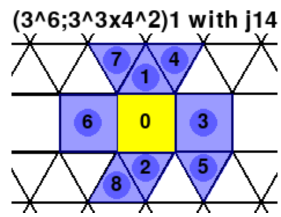
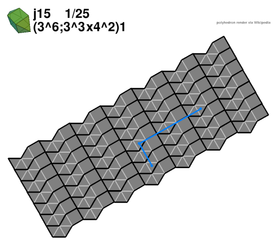
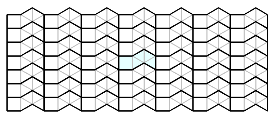
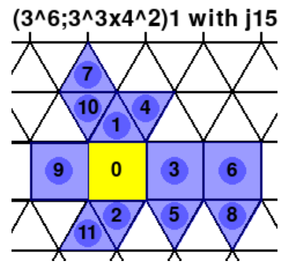
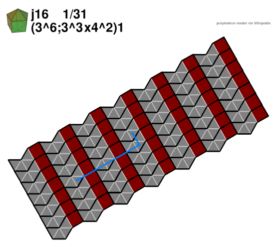
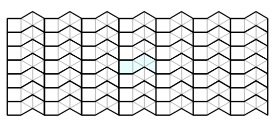
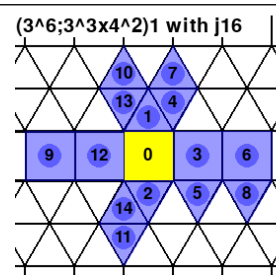
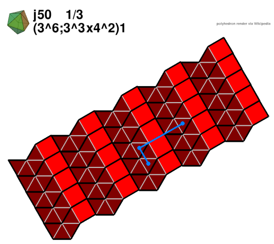
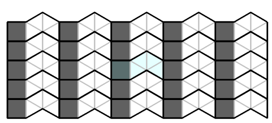
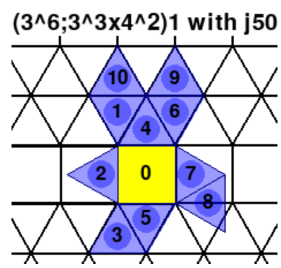
- Yellow face: example of starting face and orientation in the net
- Red face: unused face

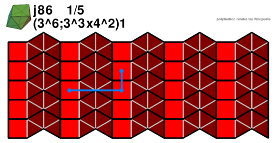
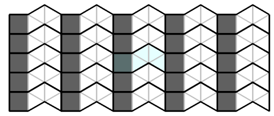
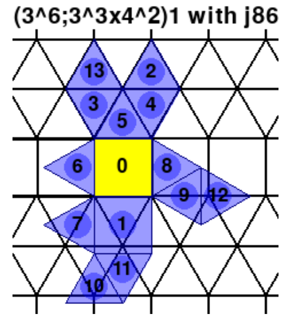
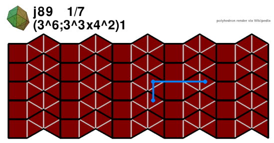
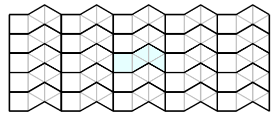
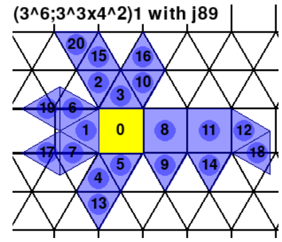
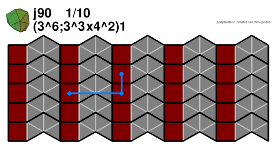
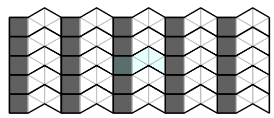
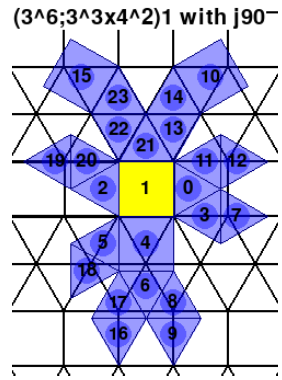
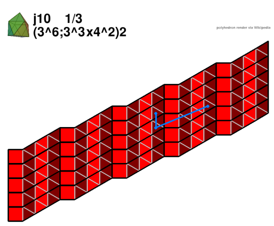
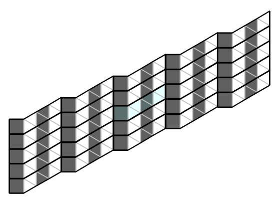
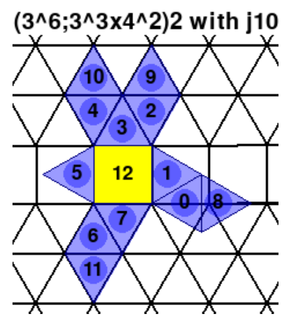
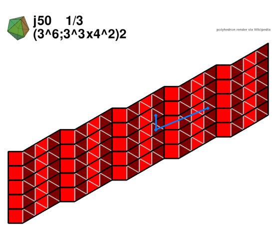
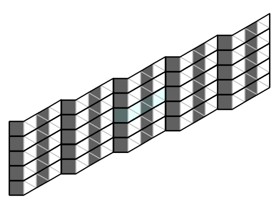
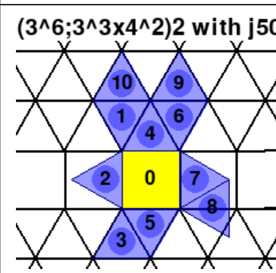
2.2 Table

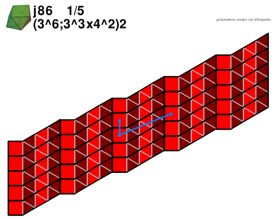
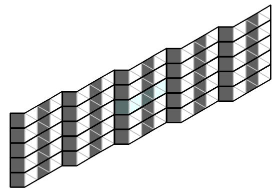
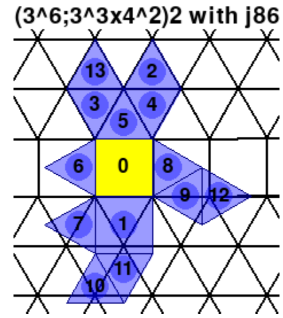
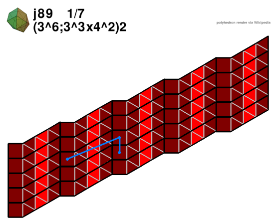
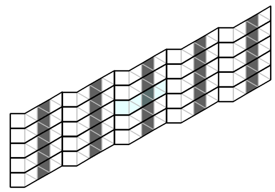
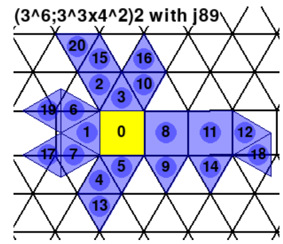
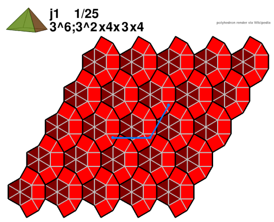
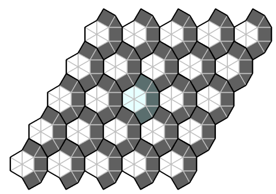
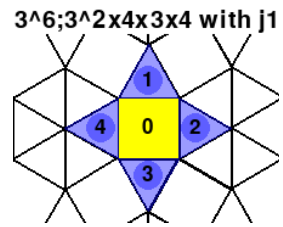
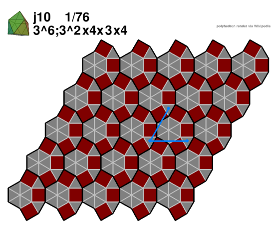
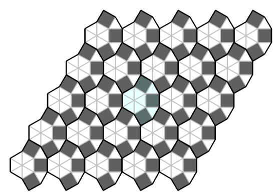
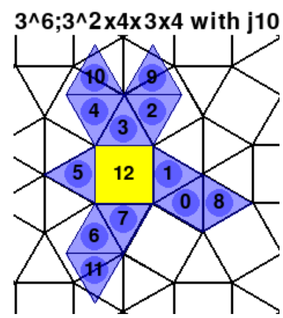
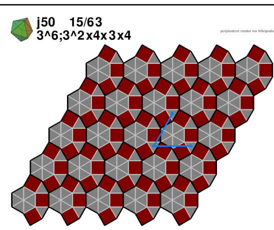
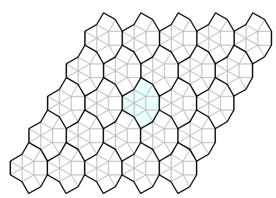
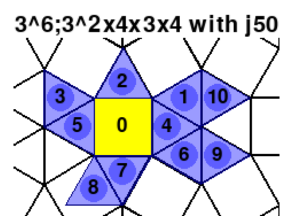
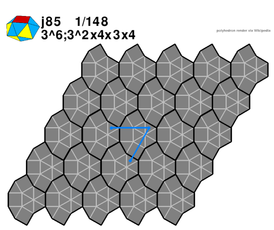
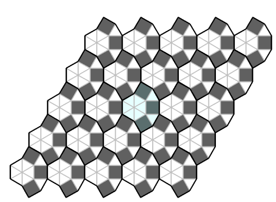
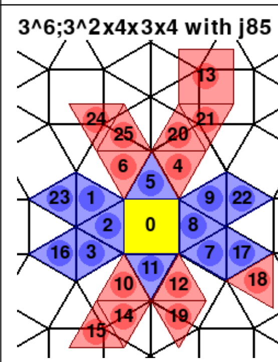
Roller Pair	Reachability	Stability	Faces
hexagonal antiprism 3^4x6			
j27 3^3x4^2			

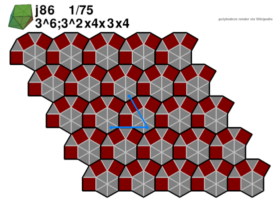
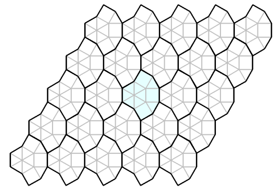
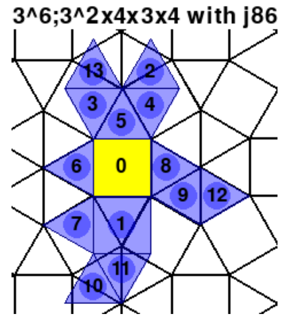
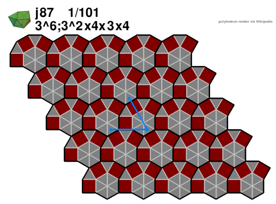
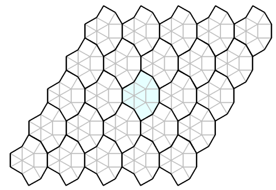
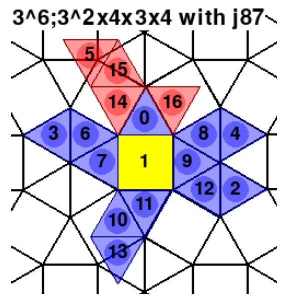
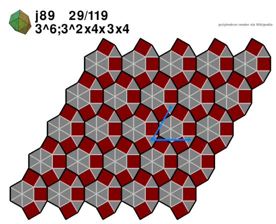
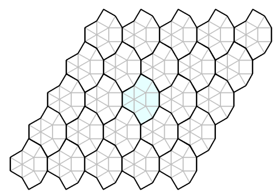
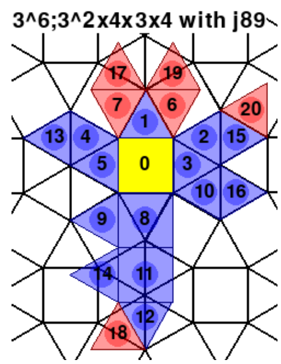
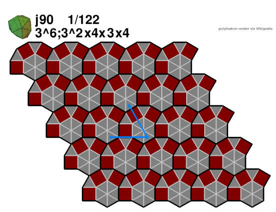
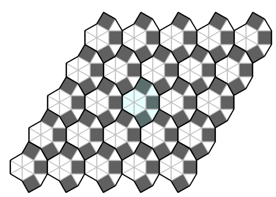
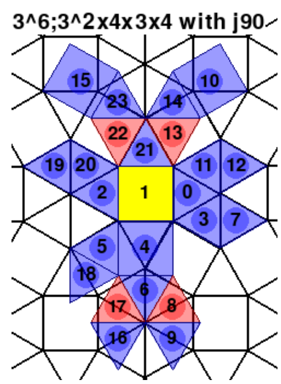
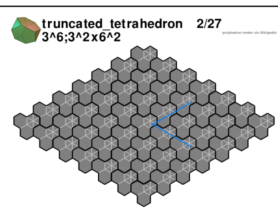
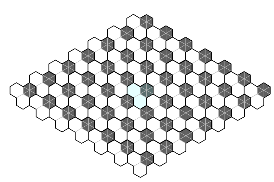
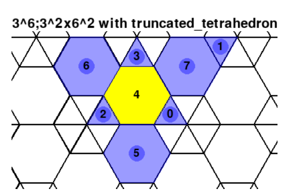
j28 3^3x4^2			
j30 3^3x4^2			
square antiprism 3^3x4^2			
j1 $3^2x4x3x4$			
j29 $3^2x4x3x4$			

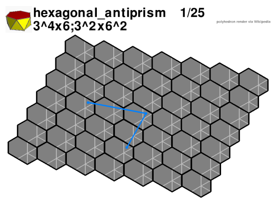
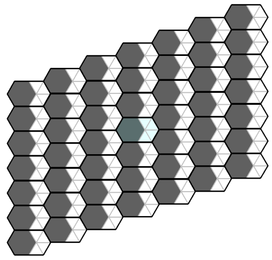
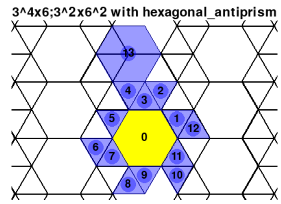
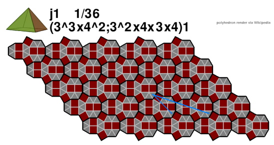
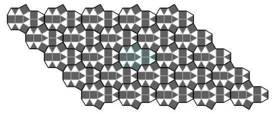
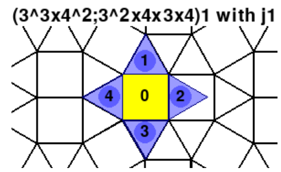
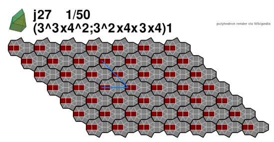
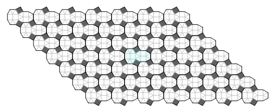
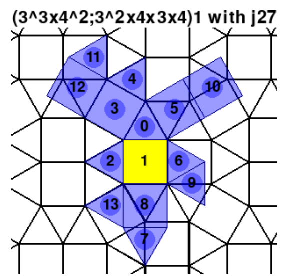
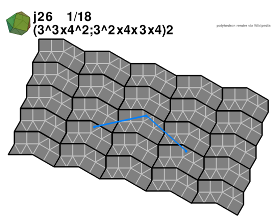
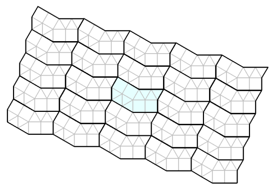
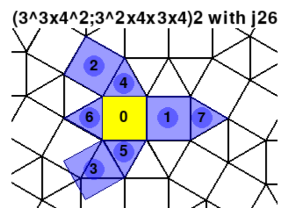
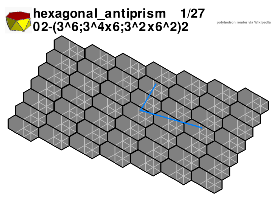
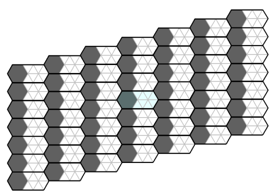
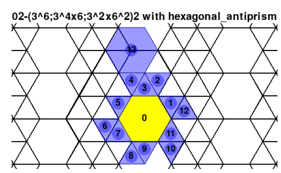
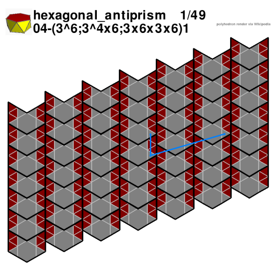
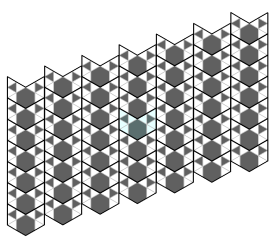
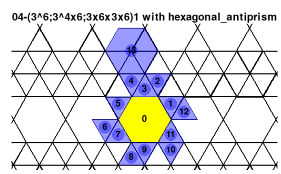
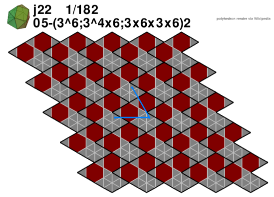
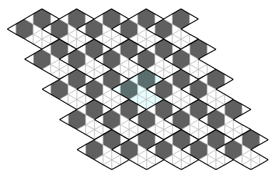
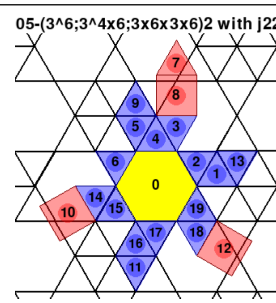
j54 $3x4x6x4$			
j56 $3x4x6x4$			
j65 $3x6x3x6$			
j22 $(3^6; 3^4x6)1$			
hexagonal antiprism $(3^6; 3^4x6)1$			

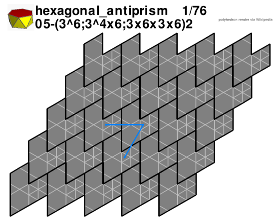
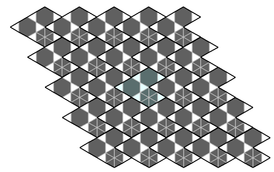
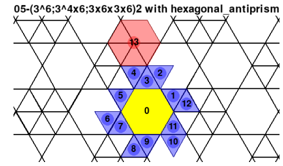
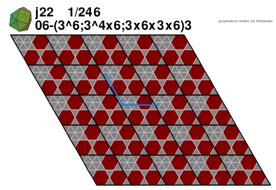
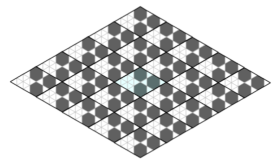
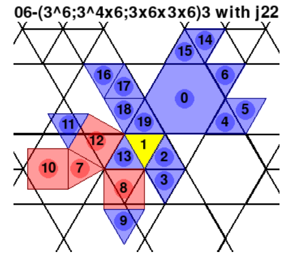
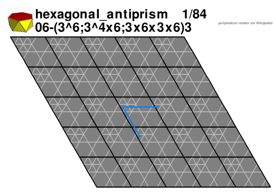
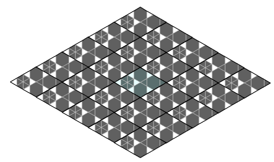
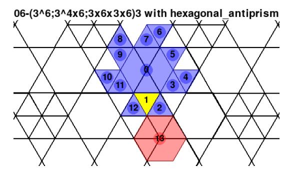
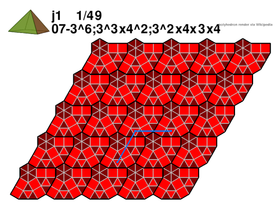
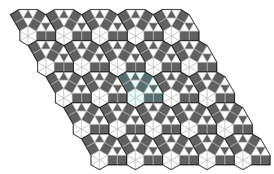
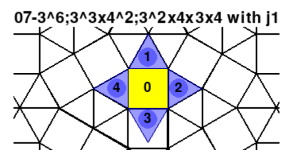
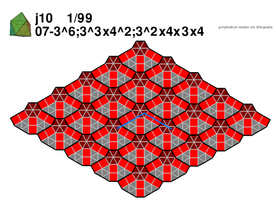
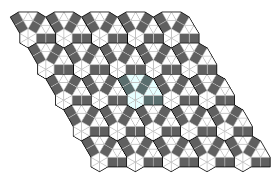
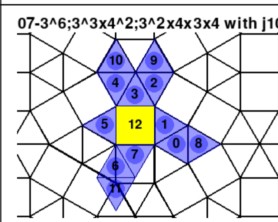
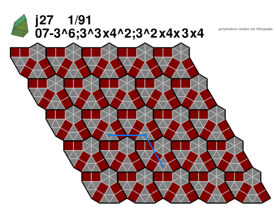
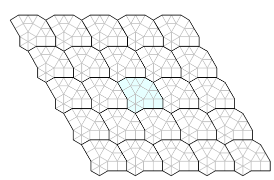
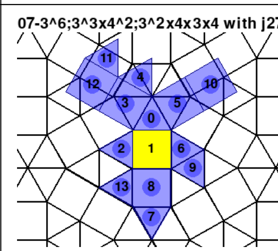
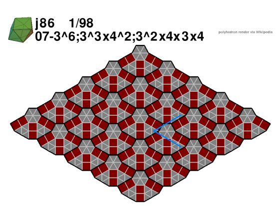
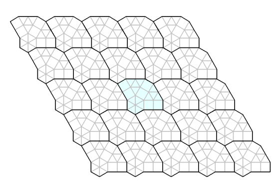
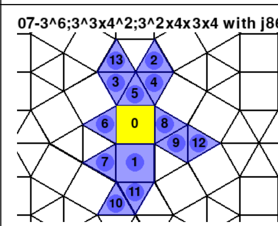
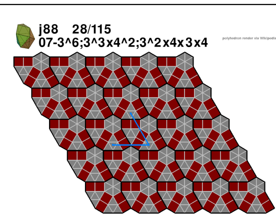
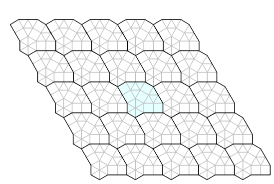
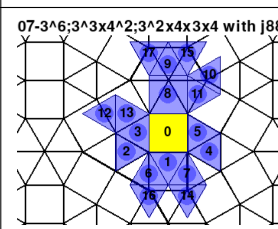
<table>
<tr><td>hexagonal antiprism
$(3^6; 3^4x6)2$</td><td></td><td></td><td></td></tr>
<tr><td>j10
$(3^6; 3^3x4^2)1$</td><td></td><td></td><td></td></tr>
<tr><td>j14
$(3^6; 3^3x4^2)1$</td><td></td><td></td><td></td></tr>
<tr><td>j15
$(3^6; 3^3x4^2)1$</td><td></td><td></td><td></td></tr>
<tr><td>j16
$(3^6; 3^3x4^2)1$</td><td></td><td></td><td></td></tr>
<tr><td>j50
$(3^6; 3^3x4^2)1$</td><td></td><td></td><td></td></tr>
</table>

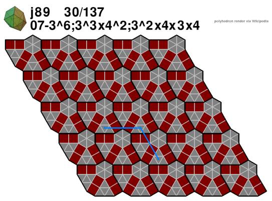
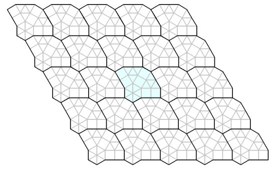
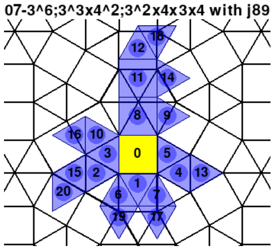
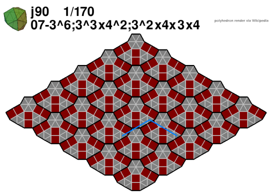
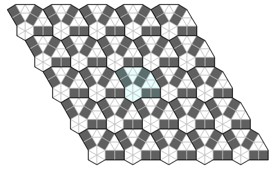
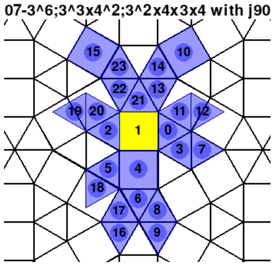
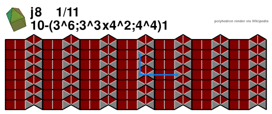
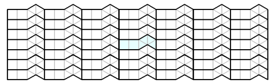
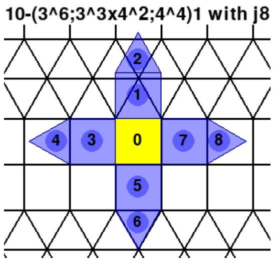
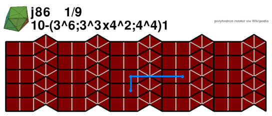
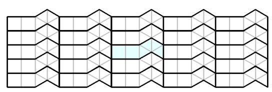
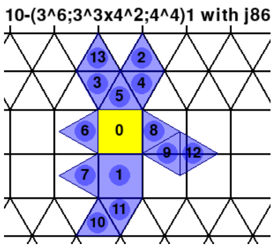
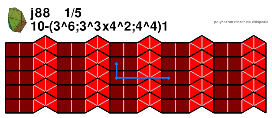
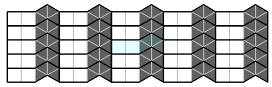
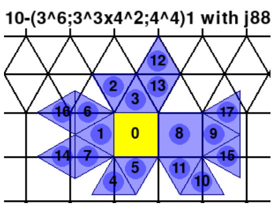
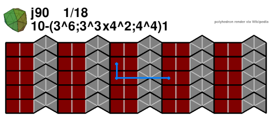
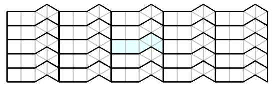
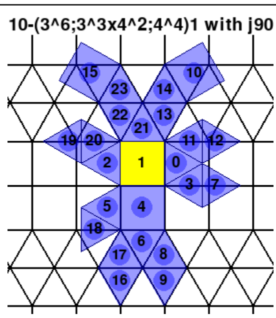
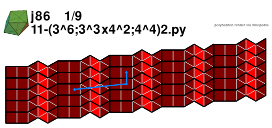
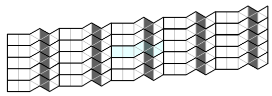
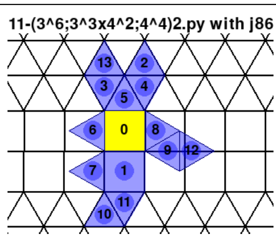
j86 $(3^6; 3^3x4^2)1$			
j89 $(3^6; 3^3x4^2)1$			
j90 $(3^6; 3^3x4^2)1$			
j10 $(3^6; 3^3x4^2)2$			
j50 $(3^6; 3^3x4^2)2$			

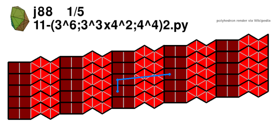
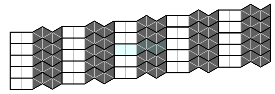
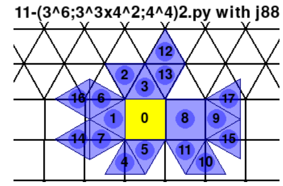
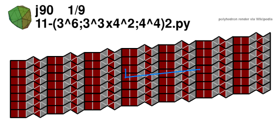
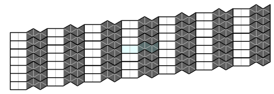
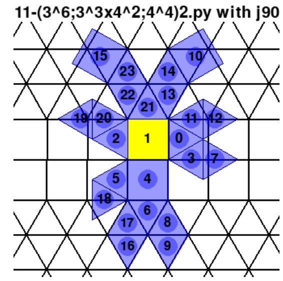
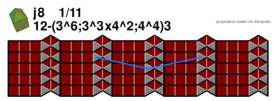
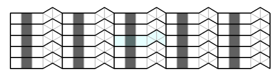
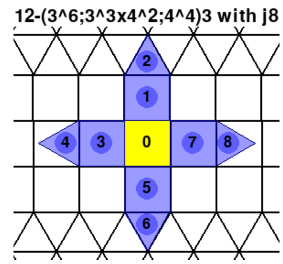
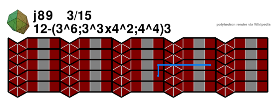
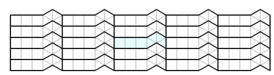
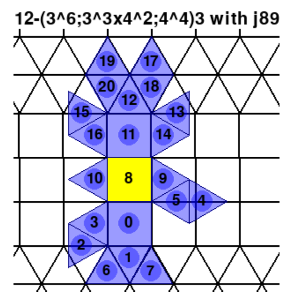
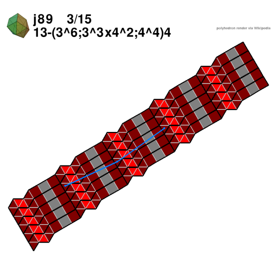
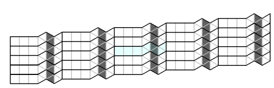
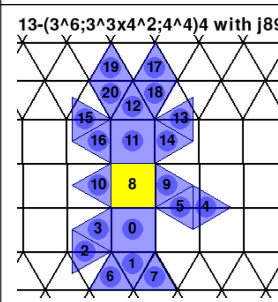
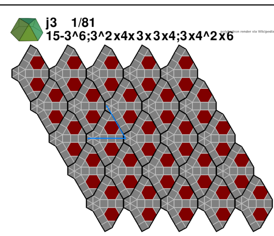
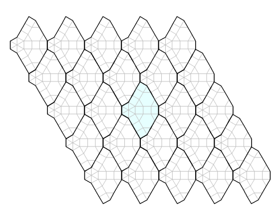
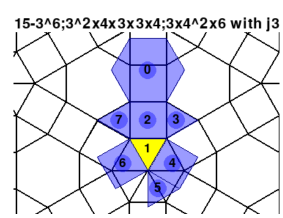
j86 $(3^6; 3^3x4^2)2$			
j89 $(3^6; 3^3x4^2)2$			
j1 $3^6; 3^2x4x3x4$			
j10 $3^6; 3^2x4x3x4$			
j50 $3^6; 3^2x4x3x4$			
j85 $3^6; 3^2x4x3x4$			

j86 $3^6; 3^2x4x3x4$			
j87 $3^6; 3^2x4x3x4$			
j89 $3^6; 3^2x4x3x4$			
j90 $3^6; 3^2x4x3x4$			
truncated tetrahedron $3^6; 3^2x6^2$			

<table>
<tr>
<td>hexagonal antiprism
$3^4x6; 3^2x6^2$</td>
<td>
</td>
<td></td>
<td>
</td>
</tr>
<tr>
<td>j1
$(3^3x4^2; 3^2x4x3x4)1$</td>
<td>
</td>
<td></td>
<td>
</td>
</tr>
<tr>
<td>j27
$(3^3x4^2; 3^2x4x3x4)1$</td>
<td>
</td>
<td></td>
<td>
</td>
</tr>
<tr>
<td>j26
$(3^3x4^2; 3^2x4x3x4)2$</td>
<td>
</td>
<td></td>
<td>
</td>
</tr>
<tr>
<td>hexagonal antiprism
$02-(3^6; 3^4x6; 3^2x6^2)2$</td>
<td>
</td>
<td></td>
<td>
</td>
</tr>
<tr>
<td>hexagonal antiprism
$04-(3^6; 3^4x6; 3x6x3x6)1$</td>
<td>
</td>
<td></td>
<td>
</td>
</tr>
<tr>
<td>j22
$05-(3^6; 3^4x6; 3x6x3x6)2$</td>
<td>
</td>
<td></td>
<td>
</td>
</tr>
</table>

<table>
<tr><td>hexagonal antiprism
$05-(3^6;3^4x6;3x6x3x6)2$</td><td></td><td></td><td></td></tr>
<tr><td>j22
$06-(3^6;3^4x6;3x6x3x6)3$</td><td></td><td></td><td></td></tr>
<tr><td>hexagonal antiprism
$06-(3^6;3^4x6;3x6x3x6)3$</td><td></td><td></td><td></td></tr>
<tr><td>j1
$07-3^6;3^3x4^2;3^2x4x3x4$</td><td></td><td></td><td></td></tr>
<tr><td>j10
$07-3^6;3^3x4^2;3^2x4x3x4$</td><td></td><td></td><td></td></tr>
<tr><td>j27
$07-3^6;3^3x4^2;3^2x4x3x4$</td><td></td><td></td><td></td></tr>
<tr><td>j86
$07-3^6;3^3x4^2;3^2x4x3x4$</td><td></td><td></td><td></td></tr>
<tr><td>j88
$07-3^6;3^3x4^2;3^2x4x3x4$</td><td></td><td></td><td></td></tr>
</table>

j89 $07-3^6;3^3x4^2;3^2x4x3x4$			
j90 $07-3^6;3^3x4^2;3^2x4x3x4$			
j8 $10-(3^6;3^3x4^2;4^4)1$			
j86 $10-(3^6;3^3x4^2;4^4)1$			
j88 $10-(3^6;3^3x4^2;4^4)1$			
j90 $10-(3^6;3^3x4^2;4^4)1$			
j86 $11-(3^6;3^3x4^2;4^4)2.py$			

j88 $11-(3^6;3^3x4^2;4^4)2.py$			
j90 $11-(3^6;3^3x4^2;4^4)2.py$			
j8 $12-(3^6;3^3x4^2;4^4)3$			
j89 $12-(3^6;3^3x4^2;4^4)3$			
j89 $13-(3^6;3^3x4^2;4^4)4$			
j3 $15-3^6;3^2x4x3x3x4;3x4^2x6$			

2.3 Highlights

J1 is an interesting shape that has many tilings it rolls. It is stable on squares (because there is onlyone square face).

3 Upcoming

- Rollers that cover the whole plane but with holes: quasi-plane rollers and hollow plane rollers
- Rollers that covers a band of the plane: band rollers
- Rollers that get stuck in an area: bounded rollers